8.7 Factoring Special Cases

* Review of 8.4:
The square of a binomial $(A+B)^2$ or $(A-B)^2$ produces a trinomial (perfect trinomial square)

$$(A+B)^2 = A^2 + 2AB + B^2$$
$$(A-B)^2 = A^2 - 2AB + B^2$$

→ How do you recognize a perfect trinomial square in order to factor it? 3 conditions must be met:

① The 1st & last terms must be squares (A^2 & B^2)

② No (-) before the A^2 & B^2

③ If you multiply the 1st & 3rd term (A & B) & then double the results you get the middle term

Examples: Is x^2+6x+9 a perfect trinomial square?

Test all 3 conditions

① 1st & 3rd terms are squares? x^2 & 9 ✓
② A^2 & B^2 are positive? x^2 & 9 ✓
③ Is 2AB the middle term? $2(x \cdot 3) = 6x$ ✓
YES

To factor: $A^2 + 2AB + B^2 = (A+B)^2$

* The sign is determined by the sign of the middle term.

$x^2 + 6x + 9 = (x+3)^2$

$x^2 - 10xy + 25y^2 = (x-5y)^2$

Factor: $27m^2 + 72mn + 48n^2$

Remember GCF 1st: $3(9m^2 + 24mn + 16n^2)$

Test 3 conditions:

① & ② $9m^2$ & $16n^2$ are perfect squares & (+) ✓

③ $2AB = 2(3m \cdot 4n) = 24mn$ ✓

Factor: $3(3m+4n)(3m+4n)$

or

$3(3m+4n)^2$

* always ✓ w/ FOIL

* Got it #1b) $x^2 - 14x + 49$

$(x-7)^2$

* Review Problem 2 on pg. 524

* Got it #2) * Remember since the area is its side length squared, factor the expression for area as the square of a binomial

$16m^2 - 72m + 81$

$(4m)^2 - 2(9m)(4) + 9^2$

$(4m-9)^2$ is the area

$(4m-9)$ is the length of one side.

* Preview of 8.4: $(A+B)(A-B) = A^2-B^2$
Binomials that differ in signs only

* Differences of Two Squares $A^2-B^2=(A+B)(A-B)$

→ How do you recognize a difference of 2 squares?
Two conditions must be met:

① 2 terms (binomial) & both terms are perfect squares
② The 2 terms are joined by a minus sign

* Got it #3)

A) V^2-100
$(V+10)(V-10)$

B) S^2-16
$(S+4)(S-4)$

* Got it #4)

A) $25d^2-64$
$(5d+8)(5d-8)$

No, it is not the difference of 2 squares, it is a sum of 2 squares

* Remember to always factor out the GCF 1st
* Got it #5)

A) $12t^2-48$
$12(t^2-4)$
$12(t-2)(t+2)$

B) $12x^2+12x+3$
$3(4x^2+4x+1)$

*Make sure always completely factor (factor until factoring is no longer possible - no other common factors other than 1)

Ex $x^8 - 1$

$(x^4+1)(x^4-1)$ ← difference of 2 squares

$(x^4+1)(x^2+1)(x^2-1)$

$(x^4+1)(x^2+1)(x+1)(x-1)$

Ex $16p^8 - t^4$

$(4p^4+t^2)(4p^4-t^2)$

$(4p^4+t^2(2p^2-t)(2p^2+t)$

Ex $3a^2b^2 - 48b^4$

$3b^2(a^2-16b^2)$

$3b^2(a+4b)(a-4b)$

Ex $-32x^2 + 18y^2$

$18y^2 - 32x^2$ * Rewrite

$2(9y^2-16x^2)$

$2(3y+4x)(3y-4x)$

Name _____ Class _____ Date _____

8-7 Practice

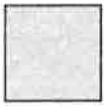
Form G

Factoring Special Cases

Factor each expression.

1. $h^2 + 10h + 25$
2. $v^2 - 14v + 49$
3. $d^2 - 22d + 121$
4. $m^2 + 4m + 4$
5. $q^2 + 6q + 9$
6. $p^2 - 24p + 144$
7. $36x^2 + 60x + 25$
8. $64x^2 + 48x + 9$
9. $49n^2 + 14n + 1$
10. $16s^2 - 72s + 81$
11. $25r^2 - 80r + 64$
12. $9g^2 - 24g + 16$
13. $81w^2 + 144w + 64$
14. $16e^2 - 88e + 121$
15. $25j^2 + 100j + 100$
16. $144f^2 - 24f + 1$
17. $4a^2 - 36a + 81$
18. $49d^2 - 84d + 36$

The given expression represents the area. Find the side length of the square.

19.  $64x^2 + 80x + 25$
20.  $9y^2 - 24y + 16$
21.  $4t^2 + 36t + 81$
22.  $36n^2 + 84n + 49$
23.  $100w^2 + 20w + 1$
24.  $16s^2 + 104s + 169$

25. **Error Analysis** Describe and correct the error made in factoring the expression at the right.

$175x^2 - 28 = 7(25x^2 - 4)$

$= 7(5x - 2)(5x - 2)$

$= 7(5x - 2)^2$

Name _____ Class _____ Date _____

8-7 Practice (continued) Form G

Factoring Special Cases

Factor each expression.

26. $m^2 - 49$ **27.** $c^2 - 100$ **28.** $p^2 - 16$

29. $4a^2 - 25$ **30.** $64n^2 - 1$ **31.** $25x^2 - 144$

32. $50g^2 - 8$ **33.** $8d^2 - 8$ **34.** $27x^2 - 48$

35. $24e^2 - 54$ **36.** $245k^2 - 20$ **37.** $112h^2 - 63$

38. $48x^2 + 72x + 27$ **39.** $8b^2 + 80b + 200$ **40.** $48w^2 + 48w + 12$

41. $45s^2 - 210s + 245$ **42.** $45t^2 - 72t + 24$ **43.** $100z^2 - 120z + 36$

44. Writing Explain how to recognize a perfect-square trinomial.

45. a. Open-Ended Write an expression that shows the factored form of a difference of two squares.

b. Explain how you know that your expression is a difference of two squares

Factor each expression.

46. $36s^8 - 60s^4 + 25$ **47.** $c^{10} - 30c^5d^2 + 225d^4$ **48.** $25n^6 + 40n^3 + 16$

Mental Math For Exercises 49–51, find a pair of factors for each number by using the difference of two squares.

49. 24 **50.** 28 **51.** 72

52. Reasoning Explain how reversing the rules for multiplying squares of binomials can help you factor a perfect-square trinomial.

53. Writing The area of a square parking lot is $49p^4 - 84p^2 + 36$. Explain how you would find the length of the parking lot.

Name Key Class _____ Date _____

8-7 Practice

Form G

Factoring Special Cases

Factor each expression.

1. $h^2 + 10h + 25$ — $(h + 5)^2$
2. $v^2 - 14v + 49$ — $(v - 7)^2$
3. $d^2 - 22d + 121$ — $(d - 11)^2$
4. $m^2 + 4m + 4$ — $(m + 2)^2$
5. $q^2 + 6q + 9$ — $(q + 3)^2$
6. $p^2 - 24p + 144$ — $(p - 12)^2$
7. $36x^2 + 60x + 25$ — $(6x + 5)^2$
8. $64x^2 + 48x + 9$ — $(8x + 3)^2$
9. $49n^2 + 14n + 1$ — $(7n + 1)^2$
10. $16s^2 - 72s + 81$ — $(4s - 9)^2$
11. $25r^2 - 80r + 64$ — $(5r - 8)^2$
12. $9g^2 - 24g + 16$ — $(3g - 4)^2$
13. $81w^2 + 144w + 64$ — $(9w + 8)^2$
14. $16e^2 - 88e + 121$ — $(4e - 11)^2$
15. $25j^2 + 100j + 100$ — $(5j + 10)^2$ $25(j+2)^2$
16. $144f^2 - 24f + 1$ — $(12f - 1)^2$
17. $4a^2 - 36a + 81$ — $(2a - 9)^2$
18. $49d^2 - 84d + 36$ — $(7d - 6)^2$

The given expression represents the area. Find the side length of the square.

19. $64x^2 + 80x + 25$ — $8x + 5$
20. $9y^2 - 24y + 16$ — $3y - 4$
21. $4t^2 + 36t + 81$ — $2t + 9$
22. $36n^2 + 84n + 49$ — $6n + 7$
23. $100w^2 + 20w + 1$ — $10w + 1$
24. $16s^2 + 104s + 169$ — $4s + 13$

25. **Error Analysis** Describe and correct the error made in factoring the expression at the right.

$(25x^2 - 4)$ factors to $(5x - 2)(5x + 2)$, not $(5x - 2)^2$

$175x^2 - 28 = 7(25x^2 - 4)$
$= 7(5x - 2)(5x - 2)$
$= 7(5x - 2)^2$

Name _____ Class _____ Date _____

8-7 Practice (continued) Form G

Factoring Special Cases

Factor each expression.

26. $m^2 - 49$
$(m + 7)(m - 7)$

27. $c^2 - 100$
$(c + 10)(c - 10)$

28. $p^2 - 16$
$(p + 4)(p - 4)$

29. $4a^2 - 25$
$(2a + 5)(2a - 5)$

30. $64n^2 - 1$
$(8n + 1)(8n - 1)$

31. $25x^2 - 144$
$(5x + 12)(5x - 12)$

32. $50g^2 - 8$
$2(5g + 2)(5g - 2)$

33. $8d^2 - 8$
$8(d + 1)(d - 1)$

34. $27x^2 - 48$
$3(3x + 4)(3x - 4)$

35. $24e^2 - 54$
$6(2e + 3)(2e - 3)$

36. $245k^2 - 20$
$5(7k + 2)(7k - 2)$

37. $112h^2 - 63$
$7(4h + 3)(4h - 3)$

38. $48x^2 + 72x + 27$
$3(4x + 3)^2$

39. $8b^2 + 80b + 200$
$8(b + 5)^2$

40. $48w^2 + 48w + 12$
$12(2w + 1)^2$

41. $45s^2 - 210s + 245$
$5(3s - 7)^2$

42. $45t^2 - 72t + 24$
$3(15t^2 - 24t + 8)$

43. $100z^2 - 120z + 36$
$4(5z - 3)^2$

44. **Writing** Explain how to recognize a perfect-square trinomial.
The coefficient of the squared term and the constant will be perfect squares. Twice the product of these numbers is the coeffiecient of the middle term.The sign before the constant will be positive.

45. **a. Open-Ended** Write an expression that shows the factored form of a difference of two squares. Answers may vary. Sample: $(2x + 3)(2x - 3)$

b. Explain how you know that your expression is a difference of two squares.
Answers may vary. Sample: $4x^2 - 9$; $4x^2$ and 9 are squares and they are separated by a subtraction.

Factor each expression.

46. $36s^8 - 60s^4 + 25$
$(6s^4 - 5)^2$

47. $c^{10} - 30c^5d^2 + 225d^4$
$(c^5 - 15d^2)^2$

48. $25n^6 + 40n^3 + 16$
$(5n^3 + 4)^2$

Mental Math For Exercises 49–51, find a pair of factors for each number by using the difference of two squares.

49. 24
$24 = 5^2 - 1^2$
$= (5 + 1)(5 - 1) = (6)(4)$

50. 28
$28 = 8^2 - 6^2$
$= (8 - 6)(8 + 6) = (2)(14)$

51. 72
$72 = 9^2 - 3^2$
$= (9 + 3)(9 - 3) = (12)(6)$

52. **Reasoning** Explain how reversing the rules for multiplying squares of binomials can help you factor a perfect-square trinomial.
When the b term in a trinomial is exactly twice the product of a and c, you can factor it as $(a + b)^2$ or as $(a - b)^2$.

53. **Writing** The area of a square parking lot is $49p^4 - 84p^2 + 36$. Explain how you would find the length of the parking lot.
Factor $49p^4 - 84p^2 + 36$ to find the length. You get $(7p^2 - 6)^2$ so each side has a length of $(7p^2 - 6)$.