8.6 Factoring Trinomials ax^2+bx+c

* 1st need to make sure the 1st term does not have a coefficient of 1

Review => Simplify $(2x+5)(3x+4)$

F O I L

$6x^2 + 8x + 15x + 20$

$6x^2 + 23x + 20$

$ax^2 + bx + c$

Set-up your quantities:

- $ax^2+bx+c = (\_x+\_)(\_x+\_)$
  * since the last sign is (+) both signs in the quantities will be the same
  * since the 1st sign is (+) both quantities will have (+) signs

- $ax^2-bx+c = (\_x-\_)(\_x-\_)$
  * since the last sign is (+) both signs in the quantities will be the same
  * since the 1st sign is (-), both quantities will have (-) signs

- $ax^2 - bx - c = (\_x + \_)(\_x - \_)$

  or

  $ax^2 + bx - c = (\_x + \_)(\_x - \_)$

* Since the last sign is (-) the signs in the quantites will be different

Steps 1) Set up your quantities
2) Find factors of ac that have a sum of b
3) Use the factors you found to rewrite bx
4) Factor out the GCF of each pair of terms
5) Rewrite with Distributive Property
6) Check with FOIL

Example:

A) Factor $2x^2 - 7x - 4$

1) $(\_x + \_)(\_x - \_)$

2)

Factors of AC ($2 \cdot -4 = -8$)	Sum of B (-7)
$1, -8$	YES
$8, -1$	NO
$2, -4$	NO
$-4, 2$	NO

3) $2x^2 + 1x - 8x - 4$

4) $(2x^2 + 1x) - (8x + 4)$
$x(2x+1) - 4(2x+1)$

5) $(x-4)(2x+1)$

6) F O I L
$x^2 + 1x - 8x - 4$
$x^2 - 7x - 4$ ✓

Answer: $(x-4)(2x+1)$

Example B:

Factor $2x^2 + 5x + 3$

1) $(\_x + \_)(\_x + \_)$

2)

Factors of AC (6)	Sum of B (5)
2, 3	yes

3) $2x^2 + 2x + 3x + 3$

4) $(2x^2 + 2x) + (3x + 3)$
$2x(x+1) + 3(x+1)$

$(2x+3)(x+1)$
check w/ FOIL ✓

Example C: Factor $6m^2+7m-5$

Factors of AC (-30)	Sum of B (7)
5, -6	NO
+10, -3	YES

$6m^2+10m-3m-5$

$2m(3m+5)-1(3m+5)$

$(2m-1)(3m+5)$

Check F O I L

$6m^2+10m-3m-5$

$6m^2+7m-5$ ✓

* Got it 1a) on pg. 519

$6x^2+13x+5$

Factors of 30	Sum of 13
6, 5	NO
10, 3	YES

$6x^2+10x+3x+5$

$2x(3x+5)+1(3x+5)$

$(2x+1)(3x+5)$ Check

F O I L

$6x^2+10x+3x+5$

$6x^2+13x+5$ ✓

* Got it 1B) Both factors are negative

* Got it 2) $10x^2+31x-14$

Factors of -140	Sum of 31
10, 14	NO
7, 20	NO
5, 28	NO
-4, 35	YES

$10x^2-4x+35x-14$

$2x(5x-2)+7(5x-2)$

$(2x+7)(5x-2)$

Check F O I L

$10x^2-4x+35x-14$

$10x^2+31x-14$

$4(2x^2-10x+1x-5)$

$4(2x(x-5)+1(x-5)$

$4(2x+1)(x-5)$

Check w/ FOIL $4(2x^2-10x+1x-5)$

$8x^2-40x+4x-20$ ✓

$8x^2-36x-20$

Name _____ Class _____ Date _____

8-6 Practice

Form G

Factoring $ax^2 + bx + c$

Factor each expression.

1. $2w^2 + 13w + 15$

2. $3d^2 + 20d + 12$

3. $4n^2 + 62n - 32$

4. $3p^2 - 7p - 40$

5. $6r^2 - 10r - 24$

6. $5z^2 - 17z + 14$

7. $14k^2 - 67k + 63$

8. $2m^2 - m - 15$

9. $3x^2 + 9x - 84$

10. $4y^2 + 26y + 30$

11. $5t^2 - 24t - 5$

12. $7c^2 - 2c - 9$

13. $8k^2 - 42k + 27$

14. $6g^2 - 2g - 20$

15. $2c^2 - 23c + 11$

16. The area of a rectangular computer screen is $4x^2 + 20x + 16$. The width of the screen is $2x + 8$. What is the length of the screen?

17. The area of a rectangular granite countertop is $12x^2 + 10x - 12$. The width of the countertop is $2x + 3$. What is the length of the countertop?

18. The area of a rectangular book cover is $4x^2 - 6x - 40$. The width of the book cover is $2x - 8$. What is the length of the book cover?

19. The area of a rectangular parking lot is $21x^2 - 44x + 15$. The width of the parking lot is $3x - 5$. What is the length of the parking lot?

Factor each expression completely.

20. $6x^2 - 10x - 4$

21. $6d^2 + 21d + 15$

22. $8n^2 + 68n + 84$

23. $20p^2 - 115p - 30$

24. $15r^2 + 141r - 90$

25. $12z^2 - 14z + 4$

26. $20k^2 + 110k + 120$

27. $9m^2 - 66m + 21$

28. $40x^2 - 136x - 96$

29. $42y^2 + 28y - 14$

30. $8t^2 - 16t - 90$

31. $24c^2 + 96c + 90$

Name _____ Class _____ Date _____

8-6 Practice

Form G

Factoring $ax^2 + bx + c$

(continued)

Open-Ended Find two different values that complete each expression so that the trinomial can be factored into the product of two binomials. Factor your trinomials.

32. $4x^2 + \square x + 12$

33. $6t^2 - \square t - 4$

34. $9m^2 - \square m + 8$

35. $8n^2 + \square n - 10$

36. $12v^2 - \square v + 15$

37. $5w^2 - \square w - 24$

38. Error Analysis Describe and correct the error made in factoring the expression at the right.

$(6x^2 + 3x - 9) = 3(2x^2 + x - 3)$
$= 3(2x^2 - 3x + 2x - 3)$
$= 3(2x^2 - 3x + (2x - 3)$
$= 3[\ x\ (2x - 3) + 1\ (2x - 3)\ ]$
$= 3(x + 1)\ (2x - 3)$

39. A parallelogram has an area of $4x^2 + 7x - 15$. The base of the parallelogram is $x + 3$. What is the height of the parallelogram?

a. Write the formula for the area of a parallelogram.

b. Writing Explain how factoring the trinomial helps you solve the problem.

40. A rectangular window pane has an area of $15x^2 - 19x + 6$. The width of the window pane is $3x - 2$. What is the length of the window pane?

Factor each expression completely.

41. $28y^2 + 43y - 48$

42. $16z^2 - 54z + 35$

43. $27n^2 - 54n + 15$

44. $36p^2 + 63p + 20$

45. $28r^2 - 20r - 33$

46. $30z^2 - 53z + 12$

47. $32x^3 + 28x^2 + 5x$

48. $25p^2 + 20pq - 12q^2$

49. $72g^2h - 43gh + 6h$

Name _____ Class _____ Date _____

Key

8-6 Practice

Form G

Factoring $ax^2 + bx + c$

Factor each expression.

1. $2w^2 + 13w + 15$
   $(2w + 3)(w + 5)$
2. $3d^2 + 20d + 12$
   $(3d + 2)(d + 6)$
3. $4n^2 + 62n - 32$
   $2(2n - 1)(n + 16)$
4. $3p^2 - 7p - 40$
   $(3p + 8)(p - 5)$
5. $6r^2 - 10r - 24$
   $2(3r + 4)(r - 3)$
6. $5z^2 - 17z + 14$
   $(5z - 7)(z - 2)$
7. $14k^2 - 67k + 63$
   $(2k - 7)(7k - 9)$
8. $2m^2 - m - 15$
   $(2m + 5)(m - 3)$
9. $3x^2 + 9x - 84$
   $3(x + 7)(x - 4)$
10. $4y^2 + 26y + 30$
    $2(2y + 3)(y + 5)$
11. $5t^2 - 24t - 5$
    $(5t + 1)(t - 5)$
12. $7c^2 - 2c - 9$
    $(7c - 9)(c + 1)$
13. $8k^2 - 42k + 27$
    $(4k - 3)(2k - 9)$
14. $6g^2 - 2g - 20$
    $2(3g + 5)(g - 2)$
15. $2c^2 - 23c + 11$
    $(2c - 1)(c - 11)$

16. The area of a rectangular computer screen is $4x^2 + 20x + 16$. The width of the screen is $2x + 8$. What is the length of the screen?
    $2x + 2$

17. The area of a rectangular granite countertop is $12x^2 + 10x - 12$. The width of the countertop is $2x + 3$. What is the length of the countertop?
    $6x - 4$

18. The area of a rectangular book cover is $4x^2 - 6x - 40$. The width of the book cover is $2x - 8$. What is the length of the book cover?
    $2x + 5$

19. The area of a rectangular parking lot is $21x^2 - 44x + 15$. The width of the parking lot is $3x - 5$. What is the length of the parking lot?
    $7x - 3$

Factor each expression completely.

20. $6x^2 - 10x - 4$
    $2(3x + 1)(x - 2)$
21. $6d^2 + 21d + 15$
    $3(2d + 5)(d + 1)$
22. $8n^2 + 68n + 84$
    $4(2n + 3)(n + 7)$
23. $20p^2 - 115p - 30$
    $5(4p + 1)(p - 6)$
24. $15r^2 + 141r - 90$
    $3(5r - 3)(r + 10)$
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    $2(2z - 1)(3z - 2)$
26. $20k^2 + 110k + 120$
    $10(2k + 3)(k + 4)$
27. $9m^2 - 66m + 21$
    $3(3m - 1)(m - 7)$
28. $40x^2 - 136x - 96$
    $8(5x + 3)(x - 4)$
29. $42y^2 + 28y - 14$
    $14(3y - 1)(y + 1)$
30. $8t^2 - 16t - 90$
    $2(2t + 5)(2t - 9)$
31. $24c^2 + 96c + 90$
    $6(2c + 5)(2c + 3)$

Name _____ Class _____ Date _____

8-6 Practice (continued) Form G

Factoring $ax^2 + bx + c$

Open-Ended **Find two different values that complete each expression so that the trinomial can be factored into the product of two binomials. Factor your trinomials.**

32. $4x^2 + \square x + 12$

Answers may vary. Sample: 19, 16: $(4x + 3)(x + 4)$; $(4x + 4)(x + 3)$

33. $6t^2 - \square t - 4$

Answers may vary. Sample: 23, −5; $(6t + 1)(t - 4)$; $(3t + 4)(2t - 1)$

34. $9m^2 - \square m + 8$

Answers may vary. Sample: 73, 27; $(9m - 1)(m - 8)$; $(3m - 8)(3m - 1)$

35. $8n^2 + \square n - 10$

Answers may vary. Sample: 11, −11; $(8n - 5)(n + 2)$; $(n - 2)(8n + 5)$

36. $12v^2 - \square v + 15$

Answers may vary. Sample: 29, 27; $(4v - 3)(3v - 5)$; $(4v - 5)(3v - 3)$

37. $5w^2 - \square w - 24$

Answers may vary. Sample. 26, 14; $(5w + 4)(w - 6)$; $(5w + 6)(w - 4)$

38. Error Analysis Describe and correct the error made in factoring the expression at the right.

$(6x^2 + 3x - 9) = 3(2x^2 + x - 3)$
$= 3(2x^2 - 3x + 2x - 3)$
$= 3(2x^2 - 3x + (2x - 3)$
$= 3[\ x(2x - 3) + 1(2x - 3)\ ]$
$= 3(x + 1)(2x - 3)$

In the second step, the student wrote $-1x$ instead of $1x$. x should be written as $3x - 2x$. Answer: $3(2x + 3)(x - 1)$

39. A parallelogram has an area of $4x^2 + 7x - 15$. The base of the parallelogram is $x + 3$. What is the height of the parallelogram?

a. Write the formula for the area of a parallelogram. $A = bh$

b. Writing Explain how factoring the trinomial helps you solve the problem.

Factor to find h: $(x + 3)(4x - 5) = 4x^2 + 7x - 15$; $h = 4x - 5$

40. A rectangular window pane has an area of $15x^2 - 19x + 6$. The width of the window pane is $3x - 2$. What is the length of the window pane?

$5x - 3$

Factor each expression completely.

41. $28y^2 + 43y - 48$
$(4y - 3)(7y + 16)$

42. $16z^2 - 54z + 35$
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$(3p + 4)(12p + 5)$

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$(2r - 3)(14r + 11)$

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$(2z - 3)(15z - 4)$

47. $32x^3 + 28x^2 + 5x$
$x(4x + 1)(8x + 5)$

48. $25p^2 + 20pq - 12q^2$
$(5p - 2q)(5p + 6q)$

49. $72g^2h - 43gh + 6h$
$h(9g - 2)(8g - 3)$