9.6 The Quadratic Formula & the Discriminant

* Reminder a quadratic equation can have 1, 2 or no real # solutions. (A quadratic equation cannot have more than 2 solutions.)

* Quadratic Formula

- best used when the other methods cannot be used
- will work for any quadratic equation

If $ax^2+bx+c=0$ and $a \neq 0$, then

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

* Review Problem 1 on pg. 583

* (Got it #1) $x^2-4x=21$

$$x^2-4x-21=0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{4 \pm \sqrt{(-4)^2-4(1)(-21)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16+84}}{2}$$

$$x = \frac{4 \pm \sqrt{100}}{2}$$

$$x = \frac{4+10}{2} \qquad x = \frac{4-10}{2}$$

$x = 7 \quad x = -3$

* Review Problem 2 on pg. 584

* Remember: sometimes the negative answer does not make sense in the given situation. Make sure your answer is always reasonable.

* (Got it #2)

$$y = -0.005x^2 + 0.7x + 3.5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-0.7 \pm \sqrt{(0.7)^2 - 4(-0.005)(3.5)}}{2(-0.005)}$$

$$x = \frac{-0.7 \pm \sqrt{0.56}}{-0.01}$$

$$x = \frac{-0.7 \pm 0.75}{-0.01}$$

$$x = -5 \text{ \& } 145$$

* negative does not make sense

145 feet

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9-6 Additional Vocabulary Support

The Quadratic Formula and the Discriminant

Complete the chart by filling in the missing information about when to use the given method to solve a quadratic equation.

Method	When to Use	Equation
completing the square	Use *completing the square* if the coefficient of x^2 is 1, but you cannot factor the equation easily. (but will work any time.)	$0 = x^2 - 2x + 5$
factoring	Use *factoring* if you can factor the equation easily.	1. $0 = x^2 + 8x + 15$ $= (x + 3)(x + 5)$
graphing	2. Use *graphing* if you have a graphing calculator available. OR you want an estimated value	$0 = 9x^2 + 12x + 4$
quadratic formula	Use the *quadratic formula* if the equation cannot be factored easily or at all. (but will work any time)	3. $0 = 2x^2 - 4x - 3$
square roots	4. Use *square* roots if the equation has no x-term.	$0 = 9x^2 - 36$

* Review Problem 3 on pg. 585 &
  Got it #3

* Discriminant - expression under the radical Sign in the quadratic formula
  - helps you determine if the quadratic equation has one, two or no real # Solutions
  - can be positive, zero or negative

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \leftarrow \text{Discriminant } (b^2 - 4ac)$$

* If the discriminant is...

  - positive, there are two real # solutions
  - negative, there are no real # solutions
  - zero, there is one real # solution

* Review the Chart on pg. 585

* Review Problem 4 on pg. 586

* Got it #4a) $6x^2 - 5x = 7$

$6x^2 - 5x - 7 = 0$

$b^2 - 4ac$

$(-5)^2 - 4(6)(-7)$

$25 + 168$

193 since the discriminat is positive there are 2 solutions

B) $b^2 - 4ac$

$b^2 - 4(a)(-c)$

$b^2 + 4a$

* must be a positive #, therefore 2 solutions

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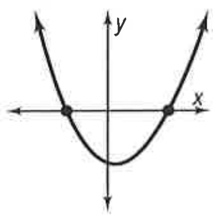
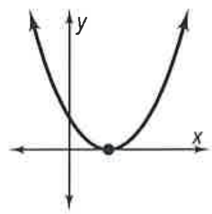
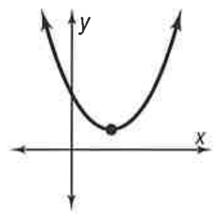
9-6 Enrichment

The Quadratic Formula and the Discriminant

You have used the discriminant to find the number of solutions to a quadratic equation. You can also use the discriminant to determine the number of x-intercepts of the graph of the related function.

Discriminant	Positive Discriminant $b^2 - 4ac > 0$	Discriminant is Zero $b^2 - 4ac = 0$	Negative Discriminant $b^2 - 4ac < 0$
Example			
Number of x-intercepts of graph of related function	The graph has two x-intercepts.	The graph has one x-intercept.	The graph has no x-intercepts.

Practice

Use the discriminant of the related quadratic equation to determine the number of x-intercepts of the graph of the function.

1. $y = x^2 + 4x + 5$
   negative, none
   $b^2 - 4ac$
   $16 - 4(1)(5)$
   $16 - 20$
   -4

2. $y = x^2 - x - 2$
   two, positive
   $b^2 - 4ac$
   $(-1)^2 - 4(1)(-2)$
   $1 + 8$
   9

3. $y = x^2 - 2x + 1$
   one, zero
   $b^2 - 4ac$
   $-2^2 - 4(1)(1)$
   $4 - 4 = 0$

4. $y = x^2 - 4x + 13$
   none, negative
   $b^2 - 4ac$
   $(-4)^2 - 4(1)(13$
   $16 - 52$
   -36

5. $y = 2x^2 + 11x - 5$
   two, positive
   $b^2 - 4ac$
   $11^2 - 4(2)(-5)$
   $121 + 40$
   161

6. $y = 4x^2 - 17x - 15$
   two, positive
   $b^2 - 4ac$
   $(-17)^2 - 4(4)(-15)$
   $289 + 240$
   529

7. $y = x^2 - 9x$
   two, positive
   $b^2 - 4ac$
   $(-9)^2 - 4(1)(0)$
   $81 - 0$
   81

8. $y = 3x^2 - 7x + 5$
   none, negative
   $b^2 - 4ac$
   $(-7)^2 - 4(3)(5)$
   $49 - 4(3)(5)$
   $49 - 60$
   -11

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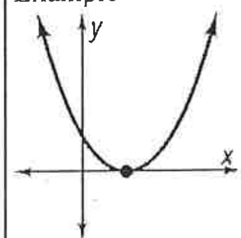
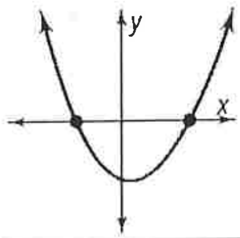
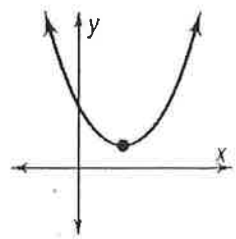
9-6 Enrichment

The Quadratic Formula and the Discriminant

You have used the discriminant to find the number of solutions to a quadratic equation. You can also use the discriminant to determine the number of x-intercepts of the graph of the related function.

Discriminant	Positive Discriminant $b^2 - 4ac > 0$	Discriminant is Zero $b^2 - 4ac = 0$	Negative Discriminant $b^2 - 4ac < 0$
Example 			
Number of x-intercepts of graph of related function	The graph has two x-intercepts.	The graph has one x-intercept.	The graph has no x-intercepts.

Practice

Use the discriminant of the related quadratic equation to determine the number of x-intercepts of the graph of the function.

1. $y = x^2 + 4x + 5$

2. $y = x^2 - x - 2$

3. $y = x^2 - 2x + 1$

4. $y = x^2 - 4x + 13$

5. $y = 2x^2 + 11x - 5$

6. $y = 4x^2 - 17x - 15$

7. $y = x^2 - 9x$

8. $y = 3x^2 - 7x + 5$

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9-6 Practice

Form K

The Quadratic Formula and the Discriminant

Use the quadratic formula to solve each equation.

1. $3z^2 + z - 4 = 0$

2. $2d^2 + 9d = 5$

3. $2y^2 + 12y + 10 = 0$

4. $2t^2 - 5t - 12 = 0$

5. $3c^2 - 13c + 4 = 0$

6. $15b^2 + 22b + 8 = 0$

Use the quadratic formula to solve each equation. Round answers to the nearest hundredth.

7. $y^2 - 4y - 4 = 0$

8. $3r^2 + 5r = 1$

9. $h^2 + 12h = -16$

10. $5v^2 + 3v = 1$

11. A football is passed through the air and caught at ground level for a touchdown. The height h of the ball in feet is given by $h = -d^2 + 12d + 6$, where d is the distance in feet the ball travels horizontally. How far from the player passing the ball will the ball be caught?

Which method(s) would you choose to solve each equation? Justify your reasoning.

12. $a^2 + 3a - 11 = 0$

13. $9d^2 - 100 = 0$

14. $6h^2 - 11h - 3 = 0$

15. $n^2 - n - 6 = 0$

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9-6 Practice (continued) *Form K*

The Quadratic Formula and the Discriminant

Find the number of real-number solutions of each equation.

16. $x^2 - 10x + 9 = 0$

17. $-5x^2 - 2x - 14 = 0$

18. $x^2 + 10x + 25 = 0$

19. $x^2 - 4x = 0$

Use the quadratic formula to solve each equation. If necessary, round answers to the nearest hundredth.

20. $4r^2 - 100 = 0$

21. $a^2 - 2a = 99$

22. $7g^2 - 2g - 10 = 0$

23. $15k^2 - 7k = 2$

Find the value of the discriminant and the number of real-number solutions of each equation.

24. $x^2 + 7x + 5 = 0$

25. $x^2 + 4x + 10 = 0$

26. $-3x^2 + 9x - 2 = 0$

27. $5x^2 + 11x + 8 = 0$

28. The daily production of a company is modeled by the function $p = -w^2 + 75w - 1200$. The daily production, p, is dependent on the number of workers, w, present. If the break-even point is when $p = 0$, what are the least and greatest number of workers the company must have present each day in order to break even?

29. **Reasoning** The equation $3x^2 + bx + 3 = 0$ has one real solution. What must be true about b?

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9-6 Practice

Form K

The Quadratic Formula and the Discriminant

Use the quadratic formula to solve each equation.

1. $3z^2 + z - 4 = 0$ $-\frac{4}{3}, 1$

2. $2d^2 + 9d = 5$ $-5, \frac{1}{2}$

3. $2y^2 + 12y + 10 = 0$ $-5, -1$

4. $2t^2 - 5t - 12 = 0$ $-\frac{3}{2}, 4$

5. $3c^2 - 13c + 4 = 0$ $\frac{1}{3}, 4$

6. $15b^2 + 22b + 8 = 0$ $-\frac{2}{3}, -\frac{4}{5}$

Use the quadratic formula to solve each equation. Round answers to the nearest hundredth.

7. $y^2 - 4y - 4 = 0$ $-0.83, 4.83$

8. $3r^2 + 5r = 1$ $-1.85, 0.18$

9. $h^2 + 12h = -16$ $-10.47, -1.53$

10. $5v^2 + 3v = 1$ $-0.84, 0.24$

11. A football is passed through the air and caught at ground level for a touchdown. The height h of the ball in feet is given by $h = -d^2 + 12d + 6$, where d is the distance in feet the ball travels horizontally. How far from the player passing the ball will the ball be caught? about 12.48 ft

Which method(s) would you choose to solve each equation? Justify your reasoning.

12. $a^2 + 3a - 11 = 0$
quadratic formula, completing the square, or graphing; the coefficient of the x^2-term is 1, but the equation cannot be factored.

13. $9d^2 - 100 = 0$
square roots; there is no x-term.

14. $6h^2 - 11h - 3 = 0$
quadratic formula, the equation cannot be factored.

15. $n^2 - n - 6 = 0$
factoring; the equation is easily factorable.

9-6 Practice (continued) — Form K

The Quadratic Formula and the Discriminant

Find the number of real-number solutions of each equation.

16. $x^2 - 10x + 9 = 0$ 2

17. $-5x^2 - 2x - 14 = 0$ no real solutions

18. $x^2 + 10x + 25 = 0$ 1

19. $x^2 - 4x = 0$ 2

Use the quadratic formula to solve each equation. If necessary, round answers to the nearest hundredth.

20. $4r^2 - 100 = 0$ ± 5

21. $a^2 - 2a = 99$ $-9, 11$

22. $7g^2 - 2g - 10 = 0$ $-1.06, 1.35$

23. $15k^2 - 7k = 2$ $-\frac{1}{5}, \frac{2}{3}$

Find the value of the discriminant and the number of real-number solutions of each equation.

24. $x^2 + 7x + 5 = 0$ 29, 2

25. $x^2 + 4x + 10 = 0$ -24; no real solutions

26. $-3x^2 + 9x - 2 = 0$ 57, 2

27. $5x^2 + 11x + 8 = 0$ -39; no real solutions

28. The daily production of a company is modeled by the function $p = -w^2 + 75w - 1200$. The daily production, p, is dependent on the number of workers, w, present. If the break-even point is when $p = 0$, what are the least and greatest number of workers the company must have present each day in order to break even? 23; 51

29. **Reasoning** The equation $3x^2 + bx + 3 = 0$ has one real solution. What must be true about b? $b = \pm 6$