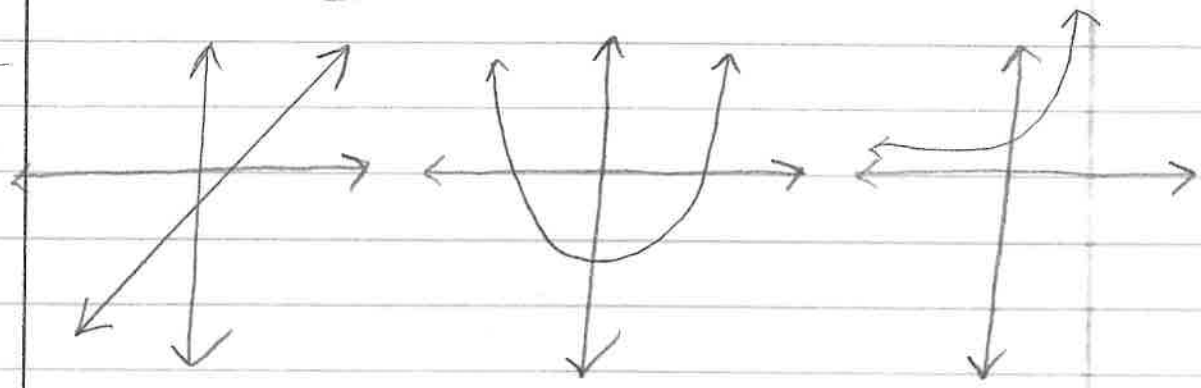
9.7 Linear, Quadratic & Exponential Models

Preview

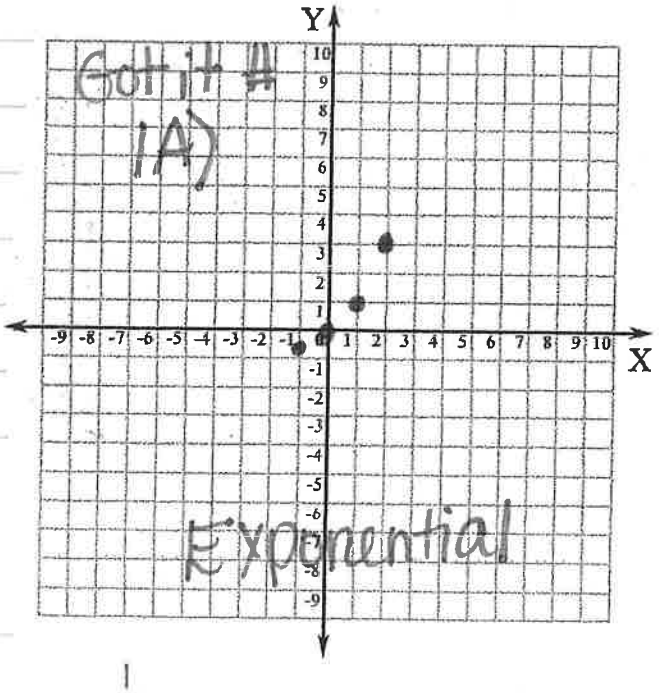


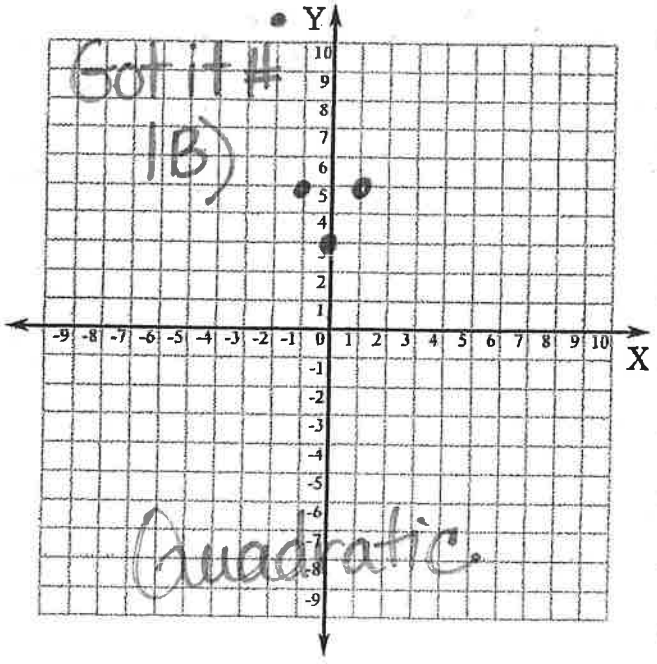
Linear
$y = mx + b$

Quadratic
$y = ax^2 + bx + c$

Exponential
$y = a \cdot b^x$

* Preview Problem 1 on pg. 589 (Choosing a Model by Graphing)





Remember:

* Linear functions

* Formula: $y = mx + b$

- y → y-coordinate (remains a variable in final form)
- m → Slope $= \frac{\Delta y}{\Delta x}$ (slope is constant, has a common difference)
- x → x-coordinate (remains a variable in final form)
- b → y-intercept $(0, y)$ (inital value)

• Can also be written as $f(x) = mx + b$

* Exponential functions

* Formula: $y = a \cdot b^x$

- y → y-coordinate (remains a variable in final form)
- a → y-intercept (inital value) $(0, y)$
- b → decay factor (common ratio of y-values)
- x → x-coordinate (remains a variable in final form)

Example: (Got it #2)

x	y	
-1	0.5	×2
0	1	×2
1	2	×2
2	4	×2
3	8	

$y = a \cdot b^x$

$y = 1 \cdot 2^x$

$y = 2^x$

NEW ⇒ Quadratic functions

* Formula:

$$y = ax^2 + bx + c$$

- y: y-coordinate (remains a variable in final form)
- a: half the second level of the common difference of y-values
- b: To find "b", use substitution
- c: y-intercept (inital value) (0, y)
- x: x-coordinates (remains a variable in final form)

Example: (Problem 2 on pg. 590)

x	y		
0	0	-0.25	-0.5
1	-0.25	-0.75	
2	-1	-1.25	-0.5
3	-2.25		
4	-4	-1.75	-0.5

*Watch your signs

* Since the 2nd level of y-values are a common difference the function is quadratic

$$y = ax^2 + bx + c$$

$$y = -\frac{1}{4}x^2 + bx + c$$

* "a" is half the common dif. $(-\frac{1}{2} \cdot \frac{1}{2} = -\frac{1}{4})$

$$y = -\frac{1}{4}x^2 + bx + 0$$

* "c" is the y-intercept $(0, y) \Rightarrow (0, 0)$

$$-1 = -\frac{1}{4}(2)^2 + b(2) + 0$$

$$-1 = -\frac{1}{4}(4) + 2b$$

$$-1 = -1 + 2b$$

$$+1 \quad +1$$

$$\frac{0 = 2b}{2}$$

$$b = 0$$

* Solve for "b" with substitution of any ordered pair (2, -1)

$$y = -\frac{1}{4}x^2$$

You Try...

Graph shows a 2nd level common difference & reflective points

①

x	y	1st diff	2nd diff
-1	1		
		-2	
0	-1		+4
		+2	
1	1		+4
		+6	
2	7		+4
		+10	
3	17		

$y = ax^2 + bx + c$

$y = 2x^2 + bx + -1$

$1 = 2(1)^2 + b(1) + -1$

$1 = 2 + 1b + -1$

$1 = 1 + 1b$

$-1 \quad -1$

$0 = b$

$y = 2x^2 - 1$

②

x	y	1st diff	2nd diff
0	1		
		+5	
1	6		+6
		+11	
2	17		+6
		+17	
3	34		+6
		+23	
4	57		+6
		+29	
5	86		

$y = ax^2 + bx + c$

$y = 3x^2 + bx + 1$

$6 = 3(1)^2 + b(1) + 1$

$6 = 3 + 1b + 1$

$6 = 4 + b$

$-4 \quad -4$

$2 = b$

$y = 3x^2 + 2x + 1$

Look for patterns in the equations...

$y = mx + b$

All constants are y-intercepts

$y = a \cdot b^x$

x & y always remain variables in final form

$y = ax^2 + bx + c$

Common ratio or differences are always effected by "x"

* Review Problem 3 on pg. 591

Name _____ Class _____ Date _____

9-7 Practice

Form G

Linear, Quadratic, and Exponential Models

Graph each set of points. Which model is most appropriate for each set?

1. (–3, –8), (–1, –2), (0, 1), (1, 4), (3, 10)

2. (–2, 0.75), (–1, 1.5), (0, 3), (1, 6)

3. (–2, 1), (–1, 0), (0, 1), (1, 4), (2, 9)

4. (–2, –11), (–1, –5), (0, –3), (1, –5), (2, –11)

5. (–4, 0), (–2, –1), (0, –2), (2, –3), (4, –4)

6. (–1, –0.67), (0, –2), (1, –6), (2, –18)

7. (–3, 10), (–1, 2), (0, 1), (1, 2), (3, 10)

8. (–2, 4), (–1, 2), (0, 0), (1, –2), (2, –4)

Which type of function best models the data in each table? Use differences or ratios.

9.

x	*y*
0	–12
1	–11
2	–8
3	–3
4	4

10.

x	*y*
0	3
1	–2
2	–7
3	–12
4	–17

11.

x	*y*
0	3
1	12
2	48
3	192
4	768

12. Which type of function best models the ordered pairs (–1, 6), (0, 1), (1, 2), and (2, 9)? Use differences or ratios.

13. Which type of function best models the ordered pairs (–1, –0.25), (0, –0.5), (1, –1), and (2, –2)? Use differences or ratios.

Name_____ Class_____ Date_____

9-7 Practice (continued) *Form G*

Linear, Quadratic, and Exponential Models

Which type of function best models the data in each table? Write an equation to model the data.

14.

x	y
0	−7
1	−1
2	5
3	11
4	17

15.

x	y
−4	32
−3	16
−2	8
−1	4
0	2

16.

x	y
0	4
1	0
2	−12
3	−32
4	−60

17.

x	y
−1	22
0	15
1	10
2	7
3	6

18.

x	y
−2	−1
−1	−2
0	−4
1	−8
2	−16

19.

x	y
0	−1
1	−2
2	−3
3	−4
4	−5

Which type of function best models the data in each ordered pair? Write an equation to model the data.

20. (−3, 33), (−1, 21), (0, 15), (1,9), (3, −3)

21. (−2, −16), (−1, −8), (0, −4), (1, −2), (2, −1)

22. $(-2, \frac{1}{27}), (-1, \frac{1}{9}), (0, \frac{1}{3}), (1, 1), (2, 3)$

23. (−2, −2), (−1, −3.5), (0, −4), (1, −3.5), (2, −2)

24. (−6, 5), (−3, 4.5), (0, 4), (3, 3.5), (6, 3)

25. (−1, 10), (0, 3), (1, 0), (2, 1)

26. The population of a city for years since 2000 is shown below. Which kind of function best models the data? Write an equation to model the data.

Years since 2000	0	2	4	6	8
Population	1500	6000	24,000	96,000	384,000

Name Key Class Date

9-7 Practice

Form G

Linear, Quadratic, and Exponential Models

Graph each set of points. Which model is most appropriate for each set?

1. (–3, –8), (–1, –2), (0, 1), (1, 4), (3, 10)

Linear
Rate of change is constant, $m = 3$

2. (–2, 0.75), (–1, 1.5), (0, 3), (1, 6)

Exponential

3. (–2, 1), (–1, 0), (0, 1), (1, 4), (2, 9)

Quadratic $(y = x^2 + 2x + 1)$

4. (–2, –11), (–1, –5), (0, –3), (1, –5), (2, –11)

Quadratic $(y = -2x^2 + 1 - 3)$

5. (–4, 0), (–2, –1), (0, –2), (2, –3), (4, –4)

Linear, $m = -\frac{1}{2}$

6. (–1, –0.67), (0, –2), (1, –6), (2, –18)

Exponential

7. (–3, 10), (–1, 2), (0, 1), (1, 2), (3, 10)

Quadratic

8. (–2, 4), (–1, 2), (0, 0), (1, –2), (2, –4)

Linear, $m = -2$

Which type of function best models the data in each table? Use differences or ratios.

9. Quadratic

x	y	
0	–12	+1
1	–11	+3
2	–8	+5
3	–3	+7
4	4	

(+2, +2, +2)

10. Linear

x	y	
0	3	–5
1	–2	–5
2	–7	–5
3	–12	–5
4	–17	

11. Exponential

x	y	
0	3	×4
1	12	×4
2	48	×4
3	192	×4
4	768	

12. Which type of function best models the ordered pairs (–1, 6), (0, 1), (1, 2), and (2, 9)? Use differences or ratios.

Quadratic (common 2nd difference)

x	y		
–1	6	–5	+6
0	1	+1	+6
1	2	+7	
2	9		

13. Which type of function best models the ordered pairs (–1, –0.25), (0, –0.5), (1, –1), and (2, –2)? Use differences or ratios.

x	y	
–1	–0.25	×2
0	–0.5	×2
1	–1	×2
2	–2	

Exponential (common ratio ×2)

$(y = 3x^2 - 2x + 1)$ See work

Name _____ Class _____ Date _____

9-7 Practice (continued)

Form G

Linear, Quadratic, and Exponential Models

Which type of function best models the data in each table? Write an equation to model the data.

14.

x	y
0	−7
1	−1
2	5
3	11
4	17

+6, +6, +6, +6

$y = 6x + -7$

Linear

15.

x	y
−4	32
−3	16
−2	8
−1	4
0	2

$\times\frac{1}{2}$, $\times\frac{1}{2}$, $\times\frac{1}{2}$, $\times\frac{1}{2}$

Exponential $y = 2 \cdot \left(\frac{1}{2}\right)^x$

16.

x	y
0	4
1	0
2	−12
3	−32
4	−60

−4, −12, −20, −28; −8, −8, −8

$a = \frac{8}{2} = 4$

$y = -1x^2 + 0x + 4$

Quadratic $y = -4x^2 + 4$

17.

x	y
−1	22
0	15
1	10
2	7
3	6

−7, −5, −3, −1; +2, +2, +2

Quadratic

$y = -x^2 + -6x + 15$

Test (2, 7)

$7 = -(2)^2 + b(2) + 15$

$7 = 4 + 2b + 15$

$7 = 19 + 2b$

$-12 = 2b$

$b = -6$

18.

x	y
−2	−1
−1	−2
0	−4
1	−8
2	−16

×2, ×2, ×2, ×2

Exponential $y = -4 \cdot 2^x$

19.

x	y
0	−1
1	−2
2	−3
3	−4
4	−5

−1, −1, −1, −1

$y = ax^2 + bx + c$

$y = 4x^2 + bx + 4$

$0 = 4(1)^2 + b(1) + 4$

$0 = 4 + 1b + 4$

$0 = 1b$

Linear $y = -1x + -1$

Which type of function best models the data in each ordered pair? Write an equation to model the data.

20. (−3, 33), (−1, 21), (0, 15), (1, 9), (3, −3)

Linear $y = -6x + 15$

21. (−2, −16), (−1, −8), (0, −4), (1, −2), (2, −1)

Exponential $y = -4 \cdot \left(\frac{1}{2}\right)^x$

22. $\left(-2, \frac{1}{27}\right)$, $\left(-1, \frac{1}{9}\right)$, $\left(0, \frac{1}{3}\right)$, (1, 1), (2, 3)

Exponential $y = \frac{1}{3} \cdot 3^x$

23. (−2, −2), (−1, −3.5), (0, −4), (1, −3.5), (2, −2)

Quadratic $y = \frac{1}{2}x^2 - 4$

24. (−6, 5), (−3, 4.5), (0, 4), (3, 3.5), (6, 3)

Linear $y = -\frac{1}{6}x + 4$

25. (−1, 10), (0, 3), (1, 0), (2, 1)

Quadratic $y = +2x^2 - 5x + 3$

see attached

26. The population of a city for years since 2000 is shown below. Which kind of function best models the data? Write an equation to model the data.

Exponential

Years since 2000	0	2	4	6	8
Population	1500	6000	24,000	96,000	384,000

×4, ×4, ×4, ×4

$y = 1500 \cdot 2^x$

$\frac{4}{2} = 2$