7-4 Division Properties of Exponents

* Dividing Powers with the same base.

$$\frac{3^5}{3^2} = \frac{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3} = 3^3$$

* What is happening with the exponents? Subtraction

* Rule: To divide powers that have the same (common) bases, subtract the exponents

* Review Problem 1 "Dividing Algebraic Expressions" on pg. 440

* Got it 1?

A) $\frac{y^{\frac{3}{4}}}{y^{\frac{1}{2}}} = y^{\frac{3}{4}-\frac{1}{2}} = y^{\frac{3}{4}-\frac{2}{4}} = y^{\frac{1}{4}}$

B) $\frac{d^{\frac{7}{2}}}{d^3} = d^{\frac{7}{2}-3} = d^{\frac{7}{2}-\frac{6}{2}} = d^{\frac{1}{2}}$

C) $\frac{K^{6} j^{2}}{K^{1} j^{5}} = K^{6-1} j^{2-5} = K^{5} j^{-3} = \frac{K^5}{j^3}$

D) $\frac{a^{-3}b^{7}}{a^{5}b^{2}} = a^{-3-5}b^{7-2} = a^{-8}b^{5} = \frac{b^{5}}{a^{8}}$

E) $\frac{x^{4}y^{-1}z^{8}}{x^{4}y^{-5}z} = x^{4-4}y^{-1-(-5)}z^{8-1} = x^{0}y^{4}z^{7} = y^{4}z^{7}$

* Review Problem 2 "Dividing Numbers in Scientific Notation" on pg. 440

* (Got it #2) $\frac{7.33 \times 10^{6}}{4.33 \times 10^{4}} = 1.69 \times 10^{2}$

about 169 people per mi^2

* Power of a Quotient

$\left(\frac{2p^{2}}{3}\right)^{4} = \frac{2^{4}(p^{2})^{4}}{3^{4}}$

* distribute the exponent to every base
* Simplify

$\frac{16p^{8}}{81}$

or

$\left(\frac{2}{3}\right)^{4} = \frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3} = \frac{2^{4}}{3^{4}} = \frac{16}{81}$

Rule: To find power of a quotient, raise the numerator & the denominator to the power & simplify

* Review Problem 3 on pg. 441

* Got it? #3

A) $\left(\frac{4}{x^3}\right)^2 = \frac{4^2}{(x^3)^2} = \frac{16}{x^6}$

B) $\left(\frac{a^{\frac{3}{4}}}{a^5}\right)^4$

→ Simplify inside the parenthese 1st

$\left(a^{\frac{3}{4}-5}\right)^4$

$\left(a^{\frac{3}{4}-\frac{20}{4}}\right)^4$

$\left(a^{-\frac{17}{4}}\right)^{\frac{4}{1}}$

$\left(a^{-17}\right) = \frac{1}{a^{17}}$

→ or find the power of the quotient 1st

$\frac{\left(a^{\frac{3}{4}}\right)^4}{(a^5)^4} = \frac{a^3}{a^{20}} = \frac{1}{a^{17}}$

* Review Problem 4, "Simplifying an Exponential Expression" on pg. 442

* Got it #4?

A) $\left(\frac{a}{5b}\right)^{-2} = \left(\frac{5b}{a}\right)^2 = \frac{25b^2}{a^2}$

⟹

* Make sure all your expressions are completely simplified. Double check to make sure...

1) Each base appears exactly once

2) There are no powers of a power

3) All fractions (as bases & exponents) are simplified

4) There are no negative EXPONENTS. (Negative factors or coefficients are fine.)

5) There are no zero exponents

6) There are no radicals

7) Variables within a term are in alphabetical order

Name _____ Class _____ Date _____

7-4 Practice

Form G

Division Properties of Exponents

Simplify each expression.

1. $\frac{5^6}{5^2}$

2. $\frac{5^5}{5^2}$

3. $\frac{x^{\frac{5}{8}}}{x^{\frac{3}{8}}}$

4. $\frac{m^{-3}}{m^{-5}}$

5. $\frac{x^6y^9}{x^2y^5}$

6. $\frac{21m^{\frac{3}{4}}}{3m^{\frac{1}{4}}}$

7. $\left(\frac{3}{5}\right)^4$

8. $\left(\frac{3x}{2y}\right)^3$

9. $\left(\frac{4}{7}\right)^{-2}$

10. $\left(-\frac{3x^4}{2y^5}\right)^{-3}$

11. $\left(\frac{12p^{\frac{3}{2}}}{15p}\right)^4$

12. $\left(\frac{ab^3}{a^5b}\right)^{-2}$

13. $\left(\frac{3x^2y^5z^{-2}}{5xz^5}\right)^{-3}$

14. $\frac{(4m^2)(3n^5)}{(2m^{-3})(-mn)^3}$

Explain why each expression is *not* in simplest form.

15. 2^4r^3

16. $(3x)^2$

17. m^3n^0

18. $\frac{y^5}{y}$

Simplify each quotient. Write each answer in scientific notation.

19. $\frac{3.6\times10^7}{1.5\times10^3}$

20. $\frac{4.5\times10^{-6}}{5\times10^{-2}}$

Practice (continued) *Form G*

Division Properties of Exponents

21. Writing Explain how you divide expressions with numerators and denominators written in scientific notation. How do you handle the exponents? What do you do with the coefficients? Connect your response to the rules you have learned regarding the division properties of exponents.

22. A computer can do a computation in 6.8×10^{-9} seconds. How many computations can the computer do in 5 minutes?

23. Error Analysis A student simplifies the expression $\left(\frac{6^4}{3^2}\right)^3$ as follows:

$\left(\frac{6^4}{3^2}\right)^3 = \left[(6 \div 3)^{4-2}\right]^3 = \left(2^2\right)^3 = 64$. What mistake did the student make in simplifying the expression? What is the correct simplification?

24. Reasoning The division property of exponents says that to simplify powers with the same base you subtract the exponents. Use examples to show why powers need to have the same base in order for this technique to work.

25. The area of a triangle is $80x^5y^3$. The height of the triangle is x^4y. What is the length of the base of the triangle?

26. Open-Ended First simplify the expression $\left(\frac{12m^5}{15m}\right)^3$ by raising each factor in the parentheses to the third power and next reducing the result. Then simplify by some other method. Explain your method. Are the results the same? Which method do you prefer?

Name Key Class _____ Date _____

7-4 Practice

Form G

Division Properties of Exponents

Simplify each expression.

1. $\frac{5^6}{5^2}$ 625 $= 5^4$

2. $\frac{5^5}{5^2}$ 125 $= 5^3$

3. $\frac{x^{\frac{5}{8}}}{x^{\frac{3}{8}}}$ $x^{\frac{1}{4}}$ $= x^{\frac{5}{8}-\frac{3}{8}} = x^{\frac{2}{8}} = x^{\frac{1}{4}}$

4. $\frac{m^{-3}}{m^{-5}}$ m^2 $= \frac{m^5}{m^3} = m^2$

5. $\frac{x^6y^9}{x^2y^5}$ x^4y^4

6. $\frac{21m^{\frac{3}{4}}}{3m^{\frac{1}{4}}}$ $7m^{\frac{1}{2}}$ $m^{\frac{3}{4}-\frac{1}{4}} = \frac{2}{4} = \frac{1}{2}$

7. $\left(\frac{3}{5}\right)^4$ $\frac{81}{625}$ $= \frac{3^4}{5^4}$

8. $\left(\frac{3x}{2y}\right)^3$ $\frac{27x^3}{8y^3}$ $\frac{3^3x^3}{2^3y^3}$

9. $\left(\frac{4}{7}\right)^{-2}$ $\frac{49}{16}$ $= \frac{7^2}{4^2}$

10. $\left(-\frac{3x^4}{2y^5}\right)^{-3}$ $\frac{8y^{15}}{27x^{12}}$ $= \frac{2^3y^{15}}{3^3x^{12}}$

11. $\left(\frac{12p^{\frac{3}{2}}}{15p}\right)^4$ $\frac{256p^2}{625}$ $= \frac{12^4p^6}{15^4p^4}$

12. $\left(\frac{ab^3}{a^5b}\right)^{-2}$ $\frac{a^8}{b^4}$ $= \frac{b^2a^{10}}{a^2b^6} = \frac{a^8}{b^4}$

13. $\left(\frac{3x^2y^5z^{-2}}{5xz^5}\right)^{-3}$ $\frac{125z^{21}}{27x^3y^{15}}$ $= \frac{5^3z^{21}}{3^3x^3y^{15}}$

14. $\frac{(4m^2)(3n^5)}{(2m^{-3})(-mn)^3}$ $-6m^2n^2$ $= \frac{12m^2n^5}{-2m^0n^3}$

Explain why each expression is *not* in simplest form.

15. 2^4r^3

2^4 is not simplified

16. $(3x)^2$

both factors should be squared

17. m^3n^0

$n^0 = 1$, so $m^3n^0 = m^3$

18. $\frac{y^5}{y}$

$\frac{y^5}{y}$ can be simplified to y^4

Simplify each quotient. Write each answer in scientific notation.

19. $\frac{3.6 \times 10^7}{1.5 \times 10^3}$ 2.4×10^4

20. $\frac{4.5 \times 10^{-6}}{5 \times 10^{-2}}$ 9.0×10^{-5} $= 0.9 \times 10^{-4}$ 9×10^{-5}

Name _____ Class _____ Date _____

7-4 Practice (continued) Form G

Division Properties of Exponents

21. Writing Explain how you divide expressions with numerators and denominators written in scientific notation. How do you handle the exponents? What do you do with the coefficients? Connect your response to the rules you have learned regarding the division properties of exponents.

For expressions with numerators and denominators in scientific notation, you work separately with the numerical parts and the powers of 10. To divide powers of 10, subtract the exponents. Put the answer back into scientific notation form.

22. A computer can do a computation in 6.8×10^{-9} seconds. How many computations can the computer do in 5 minutes? 4.41×10^{10} computers

$\frac{5 \times 60}{6.8 \times 10^{-9}} = \frac{300}{6.8 \times 10^{-9}}$

$\frac{3 \times 10^{2}}{6.8 \times 10^{-9}}$

0.441×10^{11}

23. Error Analysis A student simplifies the expression $\left(\frac{6^4}{3^2}\right)^3$ as follows: $\left(\frac{6^4}{3^2}\right)^3 = [(6 \div 3)^{4-2}]^3 = (2^2)^3 = 64$. What mistake did the student make in simplifying the expression? What is the correct simplification?

The student should have expanded the powers inside the brackets first to follow the order of operations. $\left(\frac{6 \cdot 6 \cdot 6 \cdot 6}{3 \cdot 3}\right)^3 = (2 \cdot 2 \cdot 6 \cdot 6)^3 = 144^3 = 2{,}985{,}984$

24. Reasoning The division property of exponents says that to simplify powers with the same base you subtract the exponents. Use examples to show why powers need to have the same base in order for this technique to work.

Answers may vary. Sample:

$\frac{6^5}{6^3} = \frac{6 \cdot 6 \cdot 6 \cdot 6 \cdot 6}{6 \cdot 6 \cdot 6} = 6 \cdot 6 = 36$; $\frac{6^5}{6^3} = 6^{5-3} = 6^2 = 36$; same

$\frac{6^5}{3^3} = \frac{6 \cdot 6 \cdot 6 \cdot 6 \cdot 6}{3 \cdot 3 \cdot 3} = 6 \cdot 6 \cdot 2 \cdot 2 = 288$; $\frac{6^5}{3^3} = \left(\frac{6}{3}\right)^2 = 2^2 = 4$; different

same base = same answer; different bases = different answers; The division property works only with numbers with the same bases.

25. The area of a triangle is $80x^5y^3$. The height of the triangle is x^4y. What is the length of the base of the triangle? $160xy^2$

$A = \frac{1}{2}bh$

$80x^5y^3 = \frac{1}{2}(b)(x^4y)$

$2(80x^5y^3) = \left(\frac{1}{2}bx^4y\right)2$

$\frac{160x^5y^3}{x^4y} = 160xy^2$

26. Open-Ended First simplify the expression $\left(\frac{12m^5}{15m}\right)^3$ by raising each factor in the parentheses to the third power and next reducing the result. Then simplify by some other method. Explain your method. Are the results the same? Which method do you prefer?

$\left(\frac{12m^5}{15m}\right)^3 = \frac{(12m^5)^3}{(15m)^3} = \frac{1728m^{15}}{3375m^3} = \frac{64m^{12}}{125}$; $\left(\frac{12m^5}{15m}\right)^3 = \left(\frac{4m^4}{5}\right)^3 = \frac{64m^{12}}{125}$ Simplifying within the parantheses first and then raising each factor to the thrid power gives the same results. I prefer this method because the numbers are easier to calculate.