7-7 Exponential Growth & Decay

* Exponential functions can model growth & decay

* Exponential growth
  * $y = a \cdot b^x$, where $a > 0$ & $b > 1$

  a: initial or starting amt. (when $x = 0$)

  b: growth factor or base (which is greater than 1 or 1 + % of rate of Δ)

* Review Problem 1 "Modeling Exponential Growth" on pg. 461

* Got it #1 (Part 1 only)

$y = a \cdot b^x$

$y = 285 \cdot 1.75^x$, use 9 for x (1994-1985)

$y = 285 \cdot 1.75^9$

$y \approx 43872$ subscribers

* Compound Interest -
  - example of exponential growth
  - when a bank pays interest on both the principal & interest an account has already earned

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

- A: balance
- P: principal (inital amt.)
- r: annual interest rate (decimal)
- n: # of times the interest is compounded per year
- t: time in yrs.

* Review Problem 2" Compound Interest" on pg. 462

* Got it #2? $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = 2{,}000\left(1 + \frac{0.045}{12}\right)^{12 \cdot 18}$

$A = 2{,}000(1.00375)^{216}$

$A = \$4{,}489.01$

The balance after 18 years of interest being compounded monthly is $4,489.01.

* Remember => Annually is Once a yr.
Semi-annually is 2 times a yr.
Quartly is 4 times a yr.
Monthly is 12 times a yr.

Exponential Decay

* $y = a \cdot b^x$, where $a > 0$ & $b > 1$

↓

growth factor or base (which is 1 minus % of rate of Δ.)

main difference from Exponential growth

* Review Problem 3 on pg. 463

* Got it 3

A) $y = 101 \cdot 0.885^x$

$y = 101 \cdot 0.885^5$

$y = 54.83$ or about 55 kilopascals

Name _____ Class _____ Date _____

7-7 Practice

Form G

Exponential Growth and Decay

*Worked through as a class! Not homework.

Identify the initial amount *a* and the growth factor *b* in each exponential function.

1. $f(x) = 3 \cdot 5^x$
2. $y = 250 \cdot 1.065^x$
3. $g(t) = 3.5^t$
4. $h(x) = 5 \cdot 1.02^x$

Find the balance in each account after the given period.

5. $8000 principal earning 5% compounded annually, after 6 yr
6. $2000 principal earning 5.4% compounded annually, after 4 yr
7. $500 principal earning 4% compounded quarterly, after 10 yr
8. $6500 principal earning 2.8% compounded monthly, after 2 yr

Identify the initial amount *a* and the decay factor *b* in each exponential function.

9. $y = 8 \cdot 0.8^x$
10. $f(x) = 12 \cdot 0.1^x$

State whether the equation represents *exponential growth, exponential decay,* or *neither.*

11. $y = 0.82 \cdot 3^x$
12. $f(x) = 5 \cdot 0.3^x$
13. $f(x) = 18 \cdot x^2$
14. $y = 0.9^x$

15. The town manager reports that revenue for a given year is $2.5 million. The budget director predicts that revenue will increase by 4% per yr. If the director's prediction holds true, how much revenue will the town have available 10 years from the date of the town manager's report? Write an expression to represent the equivalent monthly increase in revenue.

16. A wildlife manager determines that there are approximately 200 deer in a certain state park.
    a. The population is growing at a rate of 7% per year. How many deer will live in the park after 4 years?
    b. If the carrying capacity of this park is 350 deer, how long will it take for the deer population to reach carrying capacity?

Name _____ Class _____ Date _____

7-7 Practice (continued) Form G

Exponential Growth and Decay

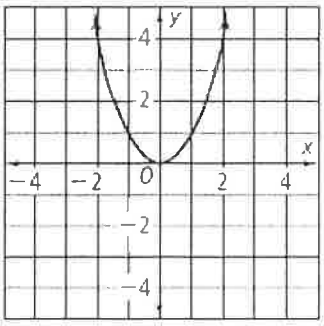
SKIP ~~17.~~ **Open-Ended** Write an exponential function that begins its rapid increase when $2 \le x \le 3$. Write another that begins its rapid increase when $3 \le x \le 4$. Write a third that begins its rapid increase when $6 \le x \le 8$.

18. A business purchases a computer system for \$3000. If the value of the system decreases at a rate of 15% per year, how much is the computer worth after 4 years?

SKIP ~~19.~~ **Writing** Explain the difference in how you would model the following situations. Person A puts \$1000 in a safe in his home, and puts in an additional \$50 per year. Person B puts \$1000 in an investment that earns 5% per year. Why is one exponential and the other linear? How would their graphs compare? How would their values compare over time?

State whether each graph shows an *exponential growth function*, an *exponential decay function*, or *neither*.

20.



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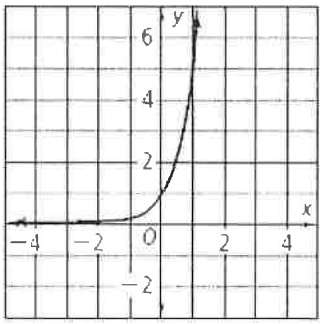

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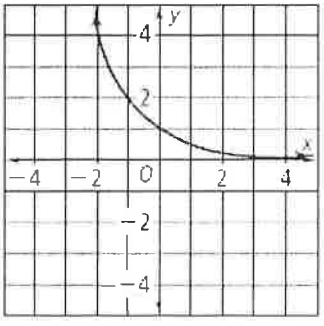

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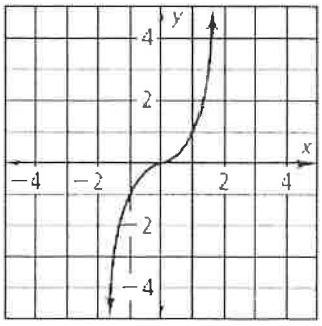

24. **Reasoning** Can the graph of an exponential function ever have a y-intercept of 0? Why or why not?

Name _____ Class _____ Date _____

7-7 Practice

Form G

Key

Exponential Growth and Decay

Identify the initial amount a and the growth factor b in each exponential function.

1. $f(x) = 3 \cdot 5^x$ $a = 3, b = 5$
2. $y = 250 \cdot 1.065^x$ $a = 250, b = 1.065$
3. $g(t) = 3.5^t$ $a = 1, b = 3.5$
4. $h(x) = 5 \cdot 1.02^x$ $a = 5, b = 1.02$

Find the balance in each account after the given period.

5. $8000 principal earning 5% compounded annually, after 6 yr $10,720.77
6. $2000 principal earning 5.4% compounded annually, after 4 yr $2468.27
7. $500 principal earning 4% compounded quarterly, after 10 yr $744.43
8. $6500 principal earning 2.8% compounded monthly, after 2 yr $6873.94

$A = 8{,}000\left(1 + \frac{0.05}{1}\right)^{1 \cdot 6}$

$A = 2{,}000\left(1 + \frac{0.054}{1}\right)^{1 \cdot 4}$

$A = 500\left(1 + \frac{0.04}{4}\right)^{4 \cdot 10}$

$A = 6500\left(1 + \frac{0.028}{12}\right)^{12 \cdot 2}$

* All exponential growth

Identify the initial amount a and the decay factor b in each exponential function.

9. $y = 8 \cdot 0.8^x$ $a = 8, b = 0.8$
10. $f(x) = 12 \cdot 0.1^x$ $a = 12, b = 0.1$

State whether the equation represents *exponential growth, exponential decay,* or *neither.*

11. $y = 0.82 \cdot 3^x$ exponential growth, $b > 1$
12. $f(x) = 5 \cdot 0.3^x$ exponential decay, $b < 1$
13. $f(x) = 18 \cdot x^2$ neither; do not know the value of x
14. $y = 0.9^x$ exponential decay, $b < 1$
15. The town manager reports that revenue for a given year is $2.5 million. The budget director predicts that revenue will increase by 4% per yr. If the director's prediction holds true, how much revenue will the town have available 10 years from the date of the town manager's report? ~~Write an expression to represent the equivalent monthly increase in revenue.~~ $3.7 million; ~~$1.0033^m$, where m is the number of months~~

$A = 2{,}500{,}000\left(1 + \frac{0.04}{1}\right)^{1 \cdot 10}$

GROWTH

16. A wildlife manager determines that there are approximately 200 deer in a certain state park.
    a. The population is growing at a rate of 7% per year. How many deer will live in the park after 4 years? approximately 262
    b. ~~If the carrying capacity of this park is 350 deer, how long will it take for the deer population to reach carrying capacity?~~ ~~between 8 and 9 years~~

$A = 200(1 + 0.07)^4$

GROWTH

Name _____ Class _____ Date _____

7-7 Practice (continued) Form G

Exponential Growth and Decay

17. **Open-Ended** Write an exponential function that begins its rapid increase when $2 \le x \le 3$. Write another that begins its rapid increase when $3 \le x \le 4$. Write a third that begins its rapid increase when $6 \le x \le 8$. Check students' work.

SKIP

18. A business purchases a computer system for $3000. If the value of the system decreases at a rate of 15% per year, how much is the computer worth after 4 years? $1566.02

DECAY

$A = 3000(1-.15)^4$

19. **Writing** Explain the difference in how you would model the following situations. Person A puts $1000 in a safe in his home, and puts in an additional $50 per year. Person B puts $1000 in an investment that earns 5% per year. Why is one exponential and the other linear? How would their graphs compare? How would their values compare over time?

For person A: $y = 1000 + 50x$, where y = total savings and x = number of years. This is linear because x and y both have a constant difference. For person B: $y = 1000(1.05)^x$, where y = total savings and x = number of years. This is an exponential function because x has a constant difference and y has a constant ratio. The graph for Person A is a straight line; the graph for Person B is an exponential curve. Person B's values will increase faster.

SKIP

State whether each graph shows an *exponential growth function*, an *exponential decay function*, or *neither*.

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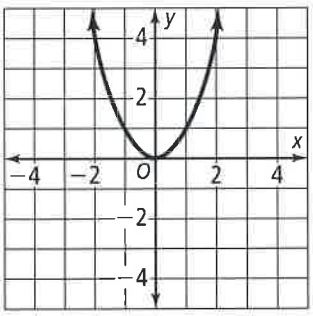

neither

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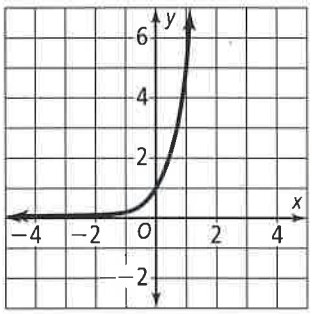
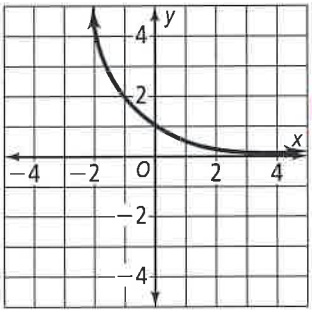


exponential growth

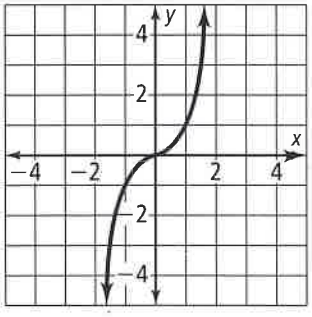
hint: * Read the graph from left to right.

22.



exponential decay

23.



neither

24. **Reasoning** Can the graph of an exponential function ever have a y-intercept of 0? Why or why not?

Answers may vary. Sample: no, because $a \neq 0$ and b^x is always positive

7-7 pg. 463 #8-20 even (#14 part only)
#27-31 all

8) Error Analysis - The student did not convert 3.5% into a decimal

$A = 500\left(1 + \frac{0.035}{4}\right)^{(4\cdot 2)}$

$A = 500(1 + 0.00875)^{8}$

$A = \$536.09$

10) $a = 150$

$b = 1.0894$

ten-thousandths

12) $a = 1$

$b = 1.4$

14) $y = a \cdot b^{x}$

$y = 100 \cdot (1.22)^{5}$

$y \approx 270$ frogs

16) $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = 12{,}000\left(1 + \frac{0.048}{1}\right)^{1\cdot 7}$

$A = 12{,}000(1 + 0.048)^{7}$

$A = \$16{,}661.35$

18) $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = 20{,}000\left(1 + \frac{0.035}{12}\right)^{12\cdot 10}$

$A = 20{,}000(1 + 0.0029)^{120}$

$A \approx \$28{,}366.90$

20) $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = 13{,}500\left(1 + \frac{0.033}{12}\right)^{12\cdot 1}$

$A = 13{,}500(1 + 0.00275)^{12}$

$A \approx \$13{,}952.30$

27) $y = a \cdot b^{x}$

$y = 45000 \cdot 0.98^{15}$

$y \approx 33{,}236$ people

28) Exponential growth b/c of 2^x, ↑1

29) Exponential decay b/c of 0.68^x, ↓1

30) Neither b/c x^2, needs to a value

31) Exponential decay b/c of 0.2^x