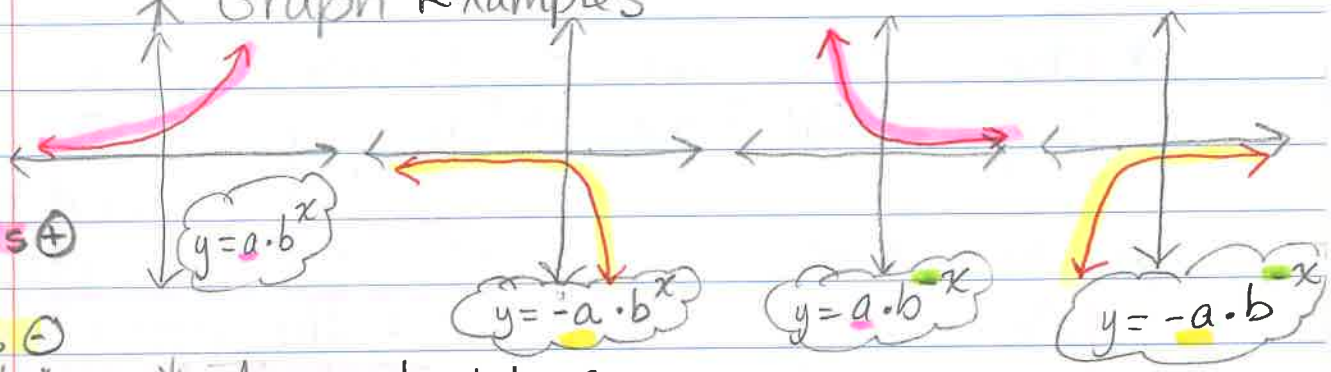
7-6 Exponential Functions

* Exponential function
  * a function where the inital amount is repeatedly multiplied by the same positive number (independent variable is an exponent)

* $y = a \cdot b^x$
  - y: dependent variable / y-coordinate / output
  - a: nonzero # / y-intercept
  - b: $b > 0$ & $b \neq 1$ (change factor)
  - x: real # / independent variable / x-coordinate / input

* Graph Examples



* negative "x"s (exponents)

* As a table:
  - x-values - have a common difference (add or subtract each value by the same #)
  - y-values - have a common ratio (multiply each value by the same #)

* Review Problem 1 "Identifying Linear & Exponential Functions" on pg. 454

* Got it 1
  A) Linear; x-values +1
  y-values +2 ⟹

B) Exponential, follows the general form $y = a \cdot b^x$

Review

* Problem 2 "Evaluating an Exponential Function on pg 454.

* Got it #2) $f(x) = 20 \cdot 3^x$

$f(x) = 20 \cdot 3^6$

$f(x) = 20 \cdot 729$

$f(x) = 14{,}580$ rabbits

$x = \frac{1}{2}$ yrs.

$2(3 = \frac{1}{2}x)$

$x = 6$

* Review Problem 3, "Graphing an Exponential Function" on pg. 455

* Remember to connect the points with a smooth curve, if the data is continuous.

* Got it #3

A)

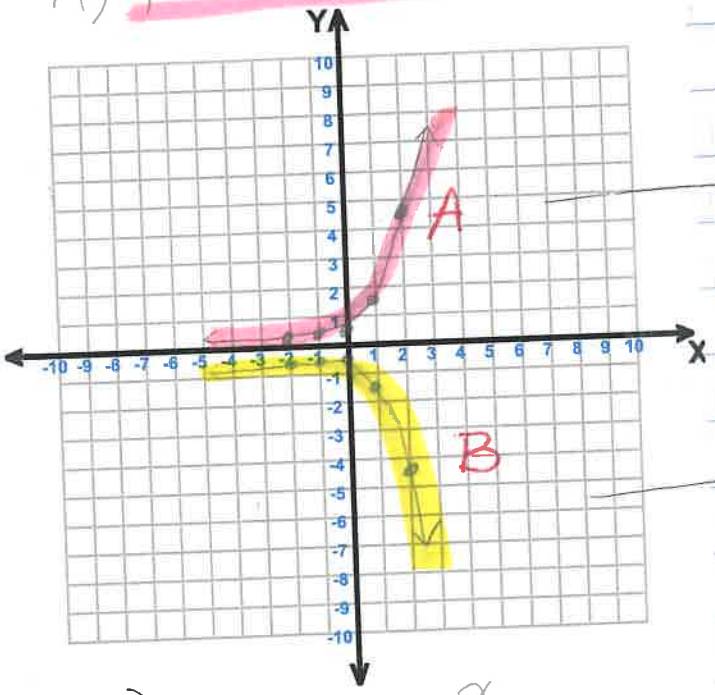
x	$y = 0.5 \cdot 3^x$	y
-2	$y = 0.5 \cdot 3^{-2}$	$+\frac{1}{18}$
-1	$y = 0.5 \cdot 3^{-1}$	$\frac{1}{6}$
0	$y = 0.5 \cdot 3^0$	$\frac{1}{2}$
1	$y = 0.5 \cdot 3^1$	$1\frac{1}{2}$
2	$y = 0.5 \cdot 3^2$	$4\frac{1}{2}$

$\Rightarrow$

B)

x	$y = -0.5 \cdot 3^x$	y
-2	$y = -0.5 \cdot 3^{-2}$	$-\frac{1}{18}$
-1	$y = -0.5 \cdot 3^{-1}$	$-\frac{1}{6}$
0	$y = -0.5 \cdot 3^{0}$	$-\frac{1}{2}$
1	$y = -0.5 \cdot 3^{1}$	$-1\frac{1}{2}$
2	$y = -0.5 \cdot 3^{2}$	$-4\frac{1}{2}$

A) $y = 0.5 \cdot 3^x$



The graph makes sense because all range values will be positive.

This graph makes sense because all range values will be negative.

B) $y = -0.5 \cdot 3^x$

* Review Problem 4, "Graphing an Exponential Model" on pg. 455

⟹

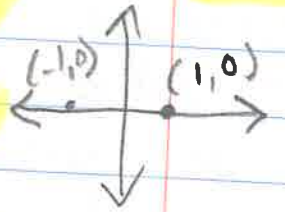
* Got it #4)

A)

x	$f(x) = 100 \cdot 4^x$	f(x)
0	$f(x) = 100 \cdot 4^0$	100
1	$f(x) = 100 \cdot 4^1$	400
2	$f(x) = 100 \cdot 4^2$	1600
3	$f(x) = 100 \cdot 4^3$	6400
4	$f(x) = 100 \cdot 4^4$	25,600

* Do not connect the data points on the graph because the answer must an intiger.

* Why does an exponential function never touch the x-axis?



* For an orderpair/line to touch the x-axis it must have a y-coordinate of zero which is impossible given the exponential function $y = a \cdot b^x$ (because that would mean either a or b^x is zero, which it cannot be)

* Given the constraint that $b > 0$, where $b \neq 0$, b^x will never $= 0$.
  - → if $x = 1$, $b^1 = b$
  - → if $x = 0$, $b^0 = 1$
  - → if $x =$ any negative, $b^{-1} = \frac{1}{b}$

  } values other than zero

* Also the constraints of a is that a is a non-zero #, therefore $y = 0 \cdot b^x$ or $y = 0$, can never exist

Name _____ Class _____ Date _____

7-6 Practice

Form K

Exponential Functions

Determine whether each table represents a linear or an exponential function. Explain. Remember that an exponential function exists when you have a constant ratio between the y values and a constant difference between the x values.

1.

x	1	2	3	4	5	6
y	2	4	8	16	32	64

2.

x	1	2	3	4	5	6
y	1	4	7	10	13	16

Determine whether each equation represents a linear or an exponential function. Remember, an exponential function takes the form $y = a \cdot b^x$ where $a \neq 0$ and $b > 0, b \neq 1$.

3. $y = 3 \cdot 2^x$

4. $y = 4 \cdot \left(\frac{1}{5}\right)^x$

5. $y = 5x - 8$

6. $y = 5 \cdot 1.07^x$

Evaluate each function for the given value. Round values to the nearest hundredth.

7. $y = 4^x$ for $x = 3$

8. $f(x) = 2 \cdot 3^x$ for $x = 5$

9. $h(t) = 60 \cdot 1.07^t$ for $t = 8$

10. $y = 5 \cdot 7^x$ for $x = 0$

11. What is the solution or solutions of $2^x = 7$? (Take an educated guess on this one. Find a value that is rounded to the nearest tenth.)

Name _____ Class _____ Date _____

7-6 Practice (continued)

Exponential Functions

Form K

12. An investment of $2000 in a bank account doubles every 5 years. The function that models the growth of this investment is $f(x) = 2000 \cdot 2^x$ where x is the number of doubling periods. How much will the investment be worth after two doubling periods, or 10 years?

13. The city library will be increasing the number of books it has to loan at a rate of 5% per year. The library currently has 40,000 books. The number of books they will have in any given year is modeled by the function $h(t) = 40{,}000 \cdot 1.05^t$ where t is the number of years. How many books will the library have 8 years from now?

Graph each exponential function.

* Use a "t" chart to find 5 ordered pairs & then graph on graph paper. (Use at least 5 "x" values. 2 positive, 2 negative & zero.) Label the ordered pairs.

14. $y = 2^x$

x	y

15. $y = 3 \cdot 2^x$

x	y

16. $y = 5^x$

x	y

17. $y = 3 \cdot 5^x$

x	y

18. **Writing** Discuss the similarities and differences between the four graphs that you sketched in Exercises 13–16.

Solve each equation.

19. $2^x = 16$

20. $10 \cdot 3^x = 90$

21. $5^x - 4 = 21$

22. $4^x + 6 = 70$

Name Key Class _____ Date _____

7-6 Practice

Form K

Exponential Functions

Determine whether each table represents a linear or an exponential function. Explain. Remember that an exponential function exists when you have a constant ratio between the y values and a constant difference between the x values.

1.

x	1	2	3	4	5	6
y	2	4	8	16	32	64

×2 ×2 ×2

Exponential (y-values have a common ratio of 2)

2.

x	1	2	3	4	5	6
y	1	4	7	10	13	16

Linear (y-values have a common difference of 3)

Determine whether each equation represents a linear or an exponential function. Remember, an exponential function takes the form $y = a \cdot b^x$ where $a \neq 0$ and $b > 0, b \neq 1$.

3. $y = 3 \cdot 2^x$ Exponential Function

4. $y = 4 \cdot \left(\frac{1}{5}\right)^x$ Exponential Function

5. $y = 5x - 8$ Linear Function

6. $y = 5 \cdot 1.07^x$ Exponential Function

Evaluate each function for the given value. Round values to the nearest hundredth.

7. $y = 4^x$ for $x = 3$ 64

8. $f(x) = 2 \cdot 3^x$ for $x = 5$ 486

9. $h(t) = 60 \cdot 1.07^t$ for $t = 8$ 103.09

10. $y = 5 \cdot 7^x$ for $x = 0$ 5

11. What is the solution or solutions of $2^x = 7$? (Take an educated guess on this one. Find a value that is rounded to the nearest tenth.)

$2^2 = 4$, $2^3 = 8$ → $2^x = 7$ → $2^{2.5} = 5.6$

$2^{2.7} = 6.49$

$2^{2.8} = 6.96$

$2^{2.9} = 7.46$

≈ 2.8

Name _____ Class _____ Date _____

7-6 Practice (continued)

Exponential Functions

Form K

$f(x) = 2{,}000 \cdot 2^x$

$f(x) = 2{,}000 \cdot 2^x$
$= 2{,}000 \cdot 2^2$
$= 8{,}000$

12. An investment of $2000 in a bank account doubles every 5 years. The function that models the growth of this investment is $f(x) = 2000 \cdot 2x$, where x is the number of doubling periods. How much will the investment be worth after two doubling periods, or 10 years?

$8,000

13. The city library will be increasing the number of books it has to loan at a rate of 5% per year. The library currently has 40,000 books. The number of books they will have in any given year is modeled by the function $h(t) = 40{,}000 \cdot 1.05^t$, where t is the number of years. How many books will the library have 8 years from now?

$h(t) = 40{,}000(1.05)^t$
$= 40{,}000(1.05)^8$
$= 59{,}098.22$

$\approx$ 59,098 books

Graph each exponential function. * Use a "t" chart to find 5 ordered pairs & then graph on graph paper. (Use at least 5 "x" values: 2 positive, 2 negative & zero.)

14. $y = 2^x$

x	y
-2	1/4
-1	1/2
0	1
1	2
2	4

15. $y = 3 \cdot 2^x$

x	y
-2	3/4
-1	3/2
0	3
1	6
2	12

* graphs attached

16. $y = 5^x$

x	y
-2	4/100
-1	0.2
0	1
1	5
2	25

17. $y = 3 \cdot 5^x$

x	y
-2	0.12
-1	0.6
0	3
1	15
2	75

18. Writing Discuss the similarities and differences between the four graphs that you sketched in Exercises 13–16.

Similarities: all graphs have positive ranges
Differences: when you multiple the term w/ the exponent the graph is steeper ($y = 5^x$ is steeper than $y = 2^x$)

Solve each equation.

19. $2^x = 16$

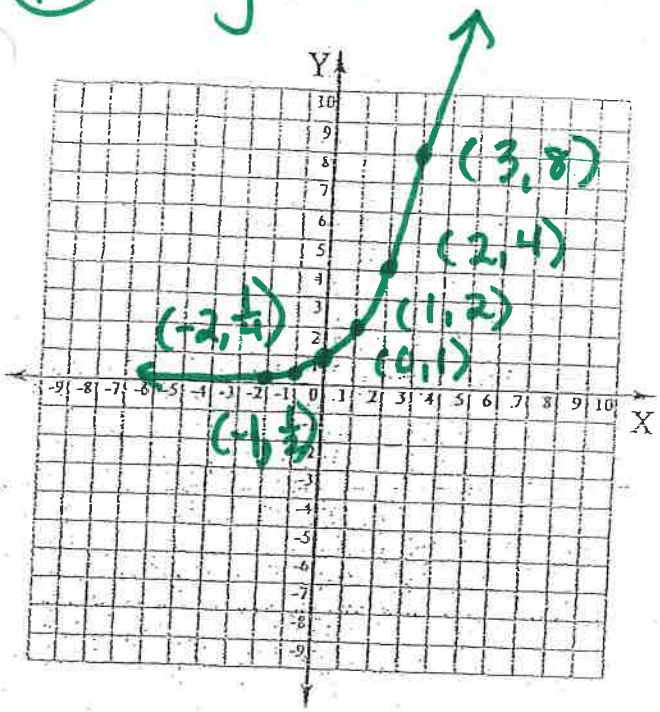
4

20. $10 \cdot 3^x = 90$
10
$3^x = 9$
$x = 2$

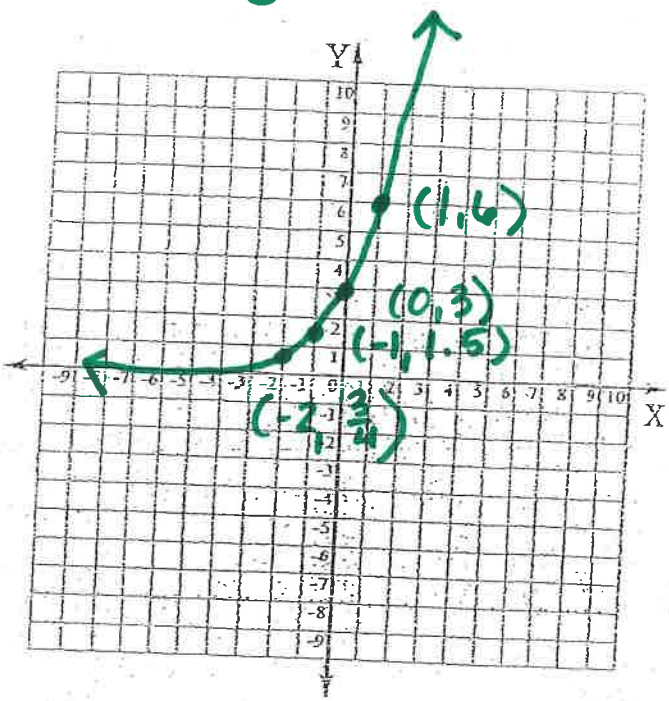
21. $5^x - 4 = 21$
$+4 \quad +4$
$5^x = 25$
$x = 2$

22. $4^x + 6 = 70$
$-6 \quad -6$
$4^x = 64$
$x = 3$

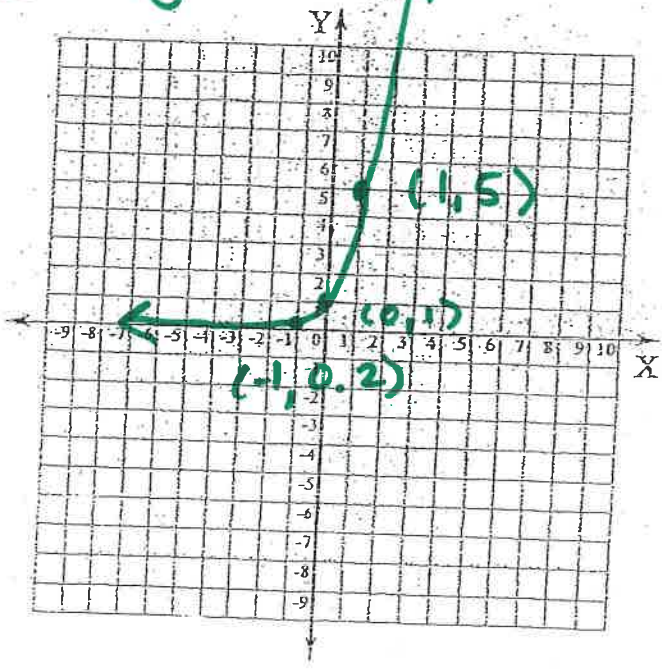
(14) $y = 2^x$



(15) $y = 3 \cdot 2^x$



(16) $y = 5^x$



(17) $y = 3 \cdot 5^x$