9.5 Completing the Square (Day 1)

* In 9.4 you were able to solve quadratic equations by finding square roots & factoring. Not all equations can be solved this way.

* Another method for solving quadratic equations is by completing the square.

* This method will work for any quadratic equation.

* Goal: Make a perfect square, so you can take the square root of each side & then solve for your variable.

* Review Problem 1 on pg. 577 & then solve

$x^2 - 16x + c$

Steps to find "c"

① Rewrite equation to $x^2 - 16x + \left(\frac{b}{2}\right)^2$

* Think about why $\left(\frac{b}{2}\right)^2 = c$ ... Since it is a perfect trinomial square, the value must be squared & you are dividing the value of b by 2 b/c the outside & inside terms double to make the middle term.

$$x^2 - 16x + \left(\frac{-16}{2}\right)^2$$

(2) Solve for "c" (Write as a square)

$$x^2 - 16x + 64$$

* Remember "c" cannot be negative. You cannot get a negative from squaring a #.

(3) Solve the equation by factoring

$$x^2 - 16x + 64 = 0$$

* Perfect Trinomial Square

$$(x-8)(x-8)$$

or

$$(x-8)^2 = 0$$

$$x = 8$$

* Got it #1) Find c & Solve

$$x^2 + 20x + c$$

$$x^2 + 20x + \left(\frac{-20}{2}\right)^2$$

$$x^2 + 20x + 100$$

$$(x+10)(x+10) = 0$$

$$x = -10$$

What if it is NOT a perfect trinomial square?

* Review Problem 2 on pg. 577

* (Got it #2) Solve for x

A) $x^2+9x+15=0$

$-15 \quad -15$

$x^2+9x=-15$

* Start to isolate "x" by moving the constant

$x^2+9x+\left(\frac{9}{2}\right)^2=-15+\left(\frac{9}{2}\right)^2$

* Add $\left(\frac{9}{2}\right)^2$ to each side of the equation (Property of Equality)

$x^2+9x+\left(\frac{9}{2}\right)^2=-15+\left(\frac{9}{2}\right)^2$

* Now I have a perfect trinomial square... it is just decimals.

$x^2+9x+20.25=-15+20.25$

$(x+4.5)^2=5.25$

$\sqrt{(x+4.5)^2}=\sqrt{5.25}$

* Round to the nearest hundredth

$x+4.5=\pm 2.29$

$x+4.5=-2.29$ $\qquad$ $x+4.5=2.29$

$x=-6.79$ $\qquad$ $x=-2.21$

$x=-6.79, -2.21$

B) No, there are no factors of 15 with a sum of 9

* Review Steps to Completing the Square

1. Rewrite the equation $x^2+bx=c$
2. Add $\left(\frac{b}{2}\right)^2$ to each side
3. write as a square
4. Take the square root of each side
5. Solve for the variable

(Day 2)

* Finding the Vertex by Completing the Square

* The equation $y=(x-h)^2+k$ represents a parabola with vertex (h,k)

* You can also use the method of Completing the Square to find the vertex.

* Review Problem 3 on pg. 577

A) $y=x^2+4x+10$

$y-10=x^2+4x$

$\left(\frac{b}{2}\right)^2=\left(\frac{4}{2}\right)^2=2^2=4$

$y-10+4=x^2+4x+4$

$y-6=(x+2)^2$

$y=(x+2)^2+6$

vertex is $(-2,6)$

* Watch Signs

B) $y = x^2 + 12x + 34$ $\quad \left(\frac{b}{2}\right)^2 = \left(\frac{12}{2}\right)^2 = 6^2 = 36$

$y - 34 + 36 = x^2 + 12x + 36$

$y + 2 = (x+6)^2$

$y = (x+6)^2 - 2$

Vertex = $(-6, -2)$

* Review Problem 4 on pg. 578

* Got it 4)

When $a \neq 1$

$(3x+2)(x+2) = 150$ → CANNOT FACTOR A GCF

$3x^2 + 8x + 4 = 150$ * Rewrite to $x^2 + bx = c$.

$-4 \quad -4$

$\frac{3x^2 + 8x = 146}{3}$ * Divide by 3 to get a coefficient of 1

$x^2 + \frac{8}{3}x = \frac{146}{3}$ $\quad * \left(\frac{b}{2}\right)^2 = \left(\frac{8}{3} \cdot \frac{1}{2}\right)^2 = \left(\frac{8}{6}\right)^2 = \frac{64}{36} = \frac{16}{9}$

$x^2 + \frac{8}{3}x + \frac{16}{9} = \frac{146}{3} + \frac{16}{9}$ $\quad 3\left(\frac{146}{3}\right) = \frac{438}{9} + \frac{16}{9}$

$\left(x + \frac{4}{3}\right)^2 = \frac{454}{9}$

$x + \frac{4}{3} = \pm 7.10$

$x + \frac{4}{3} = 7.10$ $\quad x + \frac{4}{3} = -7.10$

$x = 5.7$ ft $\quad * x = -8.43$ Negative answer does not make sense

*Make sure to round as stated in the directions

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8) $z^2 + 22z + c$

$c = \left(\frac{22}{2}\right)^2$

$c = 11^2$

$c = 121$

10) $k^2 - 5k + c$

$c = \left(\frac{-5}{2}\right)^2$

$c = 2.5^2$

$c = 6.25$ or $\frac{25}{4}$

12) $q^2 - 4q + c$

$c = \left(\frac{-4}{2}\right)^2$

$c = -2^2$

$c = 4$

14) $r^2 - 4r = 30$

$r^2 - 4r + \left(\frac{b}{2}\right)^2 = 30 + \left(\frac{b}{2}\right)^2$

$r^2 - 4r + \left(\frac{-4}{2}\right)^2 = 30 + \left(\frac{-4}{2}\right)^2$

$r^2 - 4r + 4 = 30 + 4$

$r^2 - 4r + 4 = 34$

$(r-2)^2 = 34$

$r - 2 = \pm 5.83$

$r - 2 = 5.83$ & $r - 2 = -5.83$

$+2 \quad +2$ $\quad$ $+2 \quad +2$

$r = 7.8$ $\quad$ $r = -3.8$

$r = -3.83$ & 7.83

16) $a^2 - 2a - 35 = 0$ $\quad \left(\frac{-2}{2}\right)^2 = 1$

$a^2 - 2a + 1 = 35 + 1$

$a^2 - 2a + 1 = 36$

$(a-1)^2 = 36$

$a - 1 = \pm 6$

$a - 1 = 6$ $\quad$ $a - 1 = -6$

$+1 \quad +1$ $\quad$ $+1 \quad +1$

$a = 7$ & $a = -5$

$\left(\frac{b}{2}\right)^2 = \left(\frac{-14}{2}\right)^2 = 7^2 = 49$

18) $w^2 - 14w + 13 = 0$

$w^2 - 14w + 49 = -13 + 49$

$w^2 - 14w + 49 = 36$

$(w-7)^2 = 36$

$w - 7 = \pm 6$

$w - 7 = 6$ $\quad$ $w - 7 = -6$

$+7 \quad +7$ $\quad$ $+7 \quad +7$

$w = 13$ $\quad$ & $\quad$ $w = 1$

20) $y = x^2 + 18x - 307$ $\quad \left(\frac{18}{2}\right)^2 = 9^2 = 81$

$y + 307 + 81 = x^2 + 18x + 81$

$y + 388 = (x+9)^2$

$y = (x+9)^2 - 388$

vertex $= (-9, -388)$

22) $y = x^2 + 6x - 7$ $\quad \left(\frac{b}{2}\right)^2 = \left(\frac{6}{2}\right)^2 = 3^2 = 9$

$y + 7 + 9 = x^2 + 6x + 9$

$y + 16 = (x+3)^2$

$y = (x+3)^2 - 16$ $\quad$ vertex $= (-3, -16)$

24) $y = x^2 + 12x - 468$ $\quad \left(\frac{b}{2}\right)^2 = \left(\frac{12}{2}\right)^2 = 6^2 = 36$

$y + 468 + 36 = x^2 + 12x + 36$

$y + 504 = (x+6)^2$

$y = (x+6)^2 - 504$ $\quad$ vertex $= (-6, -504)$

Factor GCF 1st

26) $2y^2-8y-10=0$

$2(y^2-4y-5)=0$

$2y^2-10y+2y-10=0$

$y^2-4y+4=5+4$

$(y-2)^2=9$

$2y(y-5)+2(y-5)=0$

$y-2=\sqrt{9}$

$(2y+2)(y-5)=0$

$y-2=\pm 3$

$2y+2=0$ $\quad$ $y-5=0$

$-2\ -2$ $\quad$ $+5\ +5$

$\frac{2y=-2}{2}$ $\quad$ $y=5$

$y=-1$

$y-2=3$ $\quad$ $y-2=-3$

$+2\ +2$ $\quad$ $+2\ +2$

$y=5$ $\quad$ $y=-1$

$\{-1,5\}$

28) $4w^2+12w-44=0$

$4(w^2+3w-11)=0$

$\left(\frac{b}{2}\right)^2=\left(\frac{3}{2}\right)^2=2.25$

GCF

$w^2+3w+2.25=11+2.25$

$(w+1.5)^2=13.25$

$w+1.5=\pm 3.64$

$w=2.14$ & -5.14

30) $2v^2 - 10v - 20 = 8$
$-8 \quad -8$

$2v^2 - 10v - 28 = 0$
$2(v^2 - 5v - 14) = 0$
$2(v^2 - 7v + 2v - 14) = 0$
$2(v(v-7) + 2(v-7)) = 0$

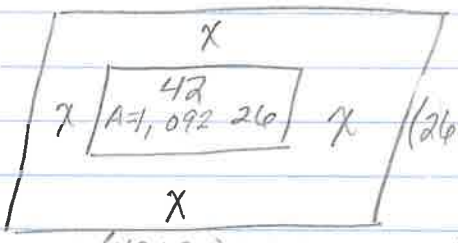
GCF

$2(v+2)(v-7) = 0$

$v = -2 \& 7$

$\left(\frac{b}{2}\right)^2 =$

32)



Area of pool
$(42)(26) = 1{,}092\ ft^2$

$(42+2x)(26+2x) = 1{,}092 + 460$
$1092 + 84x + 52x + 4x^2 = 1552$
$4x^2 + 136x = 460$
4

$x^2 + 34x = 115$
$x^2 + 34x + 289 = 115 + 289$
$(x+17)^2 = 404$

$\left(\frac{b}{2}\right)^2 = \left(\frac{34}{2}\right)^2 = 17^2 = 289$

$x + 17 = 20.1$

$x + 17 = -20.1$
$-17 \quad -17$
$x = -37.1$
* negative answer does not make sense

$x + 17 = 20.1$
$x = 3.1$ ft.

34) $q^2+3q+1=0$ $\quad \left(\frac{b}{2}\right)^2=\left(\frac{3}{2}\right)^2=(1.5)^2=2.25$

$q^2+3q=-1$

$q^2+3q+2.25=-1+2.25$

$(q+1.5)^2=+1.25$

$q+1.5=\pm 1.11$

$q=-0.39$ & -2.61

36) $w^2+7w-40=0$

$+40 \quad +40$

$w^2+7w=40$ $\quad \left(\frac{7}{2}\right)^2=3.5^2=12.25$

$w^2+7w+12.25=40+12.25$

$(w+3.5)^2=52.25$

$w+3.5=\pm 7.23$

$w=3.73$ & -10.73

38) $4p^2 - 40p + 56 = 0$

$-56 \quad -56$

$\frac{4p^2 - 40p}{4} = \frac{-56}{4}$

$\left(\frac{-10}{2}\right)^2 = (-5)^2 = 25$

$p^2 - 10p = -14$

$p^2 - 10p + 25 = -14 + 25$

$(p-5)^2 = 11$

$p - 5 = \pm 3.32$

$p = 8.32 \text{ & } 1.68$

43.) She forgot to divide each side by 4 to make the coefficient of the x^2 term a 1.

(40) $2p^2 - 15p + 8 = 43$

$-8 \quad -8$

$\frac{2p^2 - 15p}{2} = \frac{35}{2}$

$p^2 - 7.5p + \boxed{14.0625} = 17.5 + 14.0625$

$(p - 3.75)^2 = 31.5625$

$p - 3.75 = \pm 5.62$

$p - 3.75 = 5.62$ $\qquad$ $p - 3.75 = -5.62$

$p = 9.37$ $\qquad$ $p = -1.87$

(42) $s^2 + 9s + 20 = 0$

$s^2 + 9s + 20.25 = -20 + 20.25$

$(s + 4.5)^2 = \sqrt{.25}$

$s + 4.5 = \pm .5$

$s + 4.5 = .5$ or $s + 4.5 = -.5$

$s = -4$ $\qquad$ $s = -5$

(31) $A = l \cdot w$

$420 = (x)(2x + 5)$

$\frac{420}{2} = \frac{2x^2 + 5x}{2}$

$210 = x^2 + 2.5x$ $\quad +1.56$ $\quad +1.56$

$211.56 = (x + 1.25)^2$

$\pm 14.55 = x + 1.25$

about 13.3 inches

(43) $4x^2 + 10x = 8$

* She forgot to divide each side by 4 to make the coefficient of the x^2 term or "a" = 1.

(44) Too difficult to find the solutions (x-intercepts) by graphing if the solution is not an integer