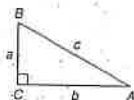**12-4** Study Guide and Intervention

The Pythagorean Theorem

The Pythagorean Theorem The side opposite the right angle in a right triangle is called the **hypotenuse**. This side is always the longest side of a right triangle. The other two sides are called the **legs** of the triangle. To find the length of any side of a right triangle, given the lengths of the other two sides, you can use the **Pythagorean Theorem**.

Pythagorean Theorem	If a and b are the measures of the legs of a right triangle and c is the measure of the hypotenuse, then $c^2 = a^2 + b^2$.	

Example 1 Find the length of the hypotenuse of a right triangle if $a = 5$ and $b = 12$.

$c^2 = a^2 + b^2$ Pythagorean Theorem
$c^2 = 5^2 + 12^2$ $a = 5$ and $b = 12$
$c^2 = 169$ Simplify.
$c = \sqrt{169}$ Take the square root of each side.
$c = 13$

The length of the hypotenuse is 13.

Example 2 Find the length of a leg of a right triangle if $a = 8$ and $c = 10$.

$c^2 = a^2 + b^2$ Pythagorean Theorem
$10^2 = 8^2 + b^2$ $a = 8$ and $c = 10$.
$100 = 64 + b^2$ Simplify.
$36 = b^2$ Subtract 64 from each side.
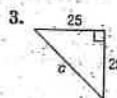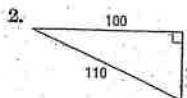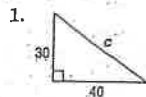$b = \pm\sqrt{36}$ Take the square root of each side.
$b = \pm 6$

The length of the leg is 6.

Exercises

Find the length of each missing side. If necessary, round to the nearest hundredth.

1.

2.

3.

If c is the measure of the hypotenuse of a right triangle, find each missing measure. If necessary, round to the nearest hundredth.

4. $a = 10, b = 12, c = ?$

5. $a = 9, b = 12, c = ?$

6. $a = 12, b = ?, c = 16$

7. $a = ?, b = 6, c = 8$

8. $a = ?, b = \sqrt{8}, c = \sqrt{18}$

9. $a = \sqrt{5}, b = \sqrt{10}, c =$

9-4 Skills Practice
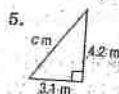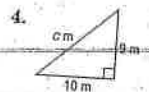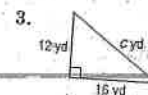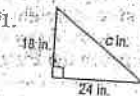
The Pythagorean Theorem

Find the length of the hypotenuse in each right triangle. Round to the nearest tenth, if necessary.

1.

2.

3.

4.

5.

6.

If c is the measure of the hypotenuse, find each missing measure. Round to the nearest tenth, if necessary.

7. $a = ?, b = 24, c = 26$

8. $a = 16, b = ?, c = 34$

9. $a = 24, b = ?, c = 40$

10. $a = 5, b = ?, c = 7$

11. $a = ?, b = 32, c = 39$

12. $a = 21, b = ?, c = 48$

13. $a = 18, b = 29, c = ?$

14. $a = ?, b = 36, c = 49$

15. $a = 8, b = ?, c = 12$

16. $a = 14, b = 21, c = ?$

17. $a = ?, b = 30, c = 40$

18. $a = 4, b = ?, c = 7$

19. $a = 13, b = 18, c = ?$

20. $a = ?, b = 55, c = 75$

The lengths of three sides of a triangle are given. Determine whether each triangle is a right triangle.

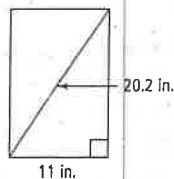21. 14 m, 5 m, 4 m

22. 3 in., 4 in., 5 in.

12-3

Close and Check

Focus Question

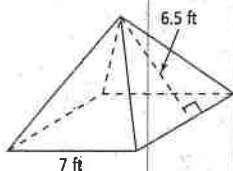When you know the lengths of two sides of a right triangle, how do you find the third?

_____

_____

_____

Do you know HOW?

1. Find the height of the sheet of paper. Round to the nearest whole number.

20.2 in.

11 in.

height = [   ] in.

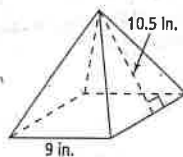**2.** Find the height of the square pyramid. Round to the nearest tenth.

6.5 ft

7 ft

height = [   ] ft

Do you UNDERSTAND?

3. Reasoning A square has a diagonal of $\sqrt{50}$. How can you find the side length of the square?

_____

_____

_____

_____

_____

4. Error Analysis Your friend found the height of the square pyramid. Explain her error.

10.5 in.

9 in.

$4.5^2 + 10.5^2 = c^2$

$130.5 = c^2$

$11.4 \approx c$
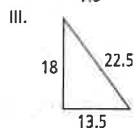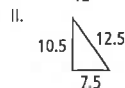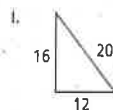
_____

_____

_____

_____

12-4

Close and Check

Focus Question

How can you determine whether a triangle is a right triangle if you do not know its angle measures and do not have measuring tools?

_____

_____

_____

_____

Do you know HOW?

1. Which triangles are right triangles?

I.

16   20

12

II.

10.5   12.5

7.5

III.

18   22.5

13.5

[   ]

2. Write an R in the box to the right of the sides lengths that form a right triangle.

A. 30, 40, 50   [   ]

B. 10, 15, 20   [   ]

C. 18, 24, 30   [   ]

D. 2, 2, 2.8   [   ]

Do you UNDERSTAND?

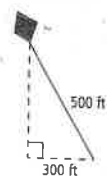**3. Reasoning** Your friend says that since the lengths 3, 4, and 5 form a right triangle, then the lengths 6, 8, and 10 must also form a right triangle. Explain why this works.

_____

_____

_____

_____

4. Writing If you have two 1-inch segments, how can you choose a third segment to make a right triangle? Is there more than one choice you can make? Explain.
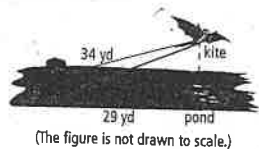
_____

_____

_____

_____

_____

1. Find the length of the third side of the right triangle. Simplify your answer.
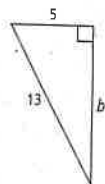
2. Tyler is flying a kite on 500 feet of string. How high is it above the ground if the horizontal distance between Tyler and the kite is 300 feet? Simplify your answer.

3. You are flying your dragon kite. It's connected to 34 yd of string. The kite is directly above the edge of a pond. The edge of the pond is 29 yd from where the kite is tied to the ground. How high is the kite above the edge of the pond? Round to one decimal place as needed.
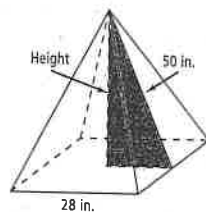

(The figure is not drawn to scale.)

4. Think About the Process A 10-foot ladder is leaning against a building with the base of the ladder 2 feet from the building.

  a. How can you find how high on the building the top of the ladder will reach?

    A. Subtract the two given values.

    B. Add the two given values.

  C. Substitute the values for a and b in the equation $a^2 + b^2 = c^2$ and solve for c.

  D. Substitute the values for a and c in the equation $a^2 + b^2 = c^2$ and solve for b.

  b. How far up the building will the top of the ladder reach?

5. Find the height of the square pyramid.

(The figure is not drawn to scale.)

6. A stainless steel patio heater is a square pyramid. The length of one side of the base is 19.8 in. The slant height of the pyramid is 92.8 in. What is the height of the pyramid? Round to one decimal place as needed.
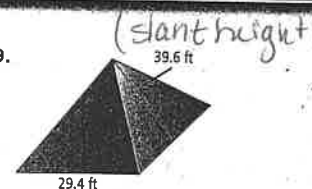
7. The base of a square pyramid is 13 ft long. Its slant height is 34.2 ft. Find the height of the square pyramid. Round to one decimal place as needed.

8.

diagonal is 325ft

(The figure is not drawn to scale.)

  a. Writing What is the length of the rectangular plot of land shown?

 **b.** How are the lengths of the legs of a right triangle related to the lengths of the sides of a rectangle?

9.

(slant height)
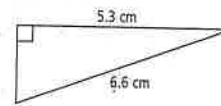
(The figure is not drawn to scale.)

  a. Reasoning What is the height of the square pyramid? Round to one decimal place as needed.

  b. Once you know which length represents the hypotenuse, does it matter which length you substitute for a and which length you substitute for b? Explain.

10. Find the length of the unknown leg of the right triangle.
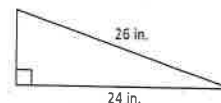
(The figure is not drawn to scale.)

11. Error Analysis A student was asked to find the length of the unknown leg of the right triangle. She incorrectly said that the length of the unknown leg of the right triangle is about 6.2 cm.

(The figure is not drawn to scale.)

  a. Find the length of the unknown leg of the right triangle. Round to one decimal place as needed.

  b. What mistake might the student have made?

12. Camping A guy rope is attached to the top of a tent pole. The guy rope is pegged into the ground 6 feet from the tent. If the guy rope is 11 feet long, how long is the tent pole? Round to one decimal place as needed.

13. The area of the base of a square pyramid is 100 cm². Its slant height is 36.8 cm. Find the height of the square pyramid. Round to one decimal place as needed.

14. Think About the Process A lamp base is a square pyramid. One side of the base is 13 cm long. The slant height is 35.6 cm.

  a. What is the first step in finding the height of the square pyramid?

    A. Substitute the given values into $a^2 + b^2 = c^2$.
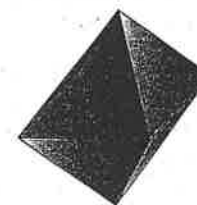
    B. Divide the length of one side of the base by 2.

    C. Add the two given values.
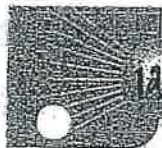
    D. Subtract the two given values.

  b. Find the height of the square pyramid. Round to one decimal place as needed.

15. Challenge To get to a cabin, Diana can bike west from a parking lot along the edge of a rectangular reservoir for 0.5 miles and then south along the edge. Alternatively, she can row a boat directly from the parking lot for 1.3 miles. If Diana can ride 1.4 times as fast as she can row, which method of travel will get Diana to the cabin faster?

16. Challenge An 8-sided game piece has the shape of two identical square pyramids attached at their bases. The perimeter of the square is 80 mm and the slant height of each pyramid is 17 mm. What is the distance from the apex of one pyramid to the apex of the other? Round to one decimal place as needed.

skip

Key (handwritten)

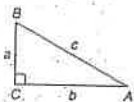Digits 12.4 (handwritten)

12-4 Study Guide and Intervention
The Pythagorean Theorem

The Pythagorean Theorem The side opposite the right angle in a right triangle is called the **hypotenuse**. This side is always the longest side of a right triangle. The other two sides are called the **legs** of the triangle. To find the length of any side of a right triangle, given the lengths of the other two sides, you can use the Pythagorean Theorem.

| Pythagorean Theorem | If a and b are the measures of the legs of a right triangle and c is the measure of the hypotenuse, then $c^2 = a^2 + b^2$. | |

Example 1 Find the length of the hypotenuse of a right triangle if $a = 5$ and $b = 12$.

$c^2 = a^2 + b^2$ Pythagorean Theorem
$c^2 = 5^2 + 12^2$ $a = 5$ and $b = 12$
$c^2 = 169$ Simplify.
$c = \sqrt{169}$ Take the square root of each side.
$c = 13$

The length of the hypotenuse is 13.

Example 2 Find the length of a leg of a right triangle if $a = 8$ and $c = 10$.

$c^2 = a^2 + b^2$ Pythagorean Theorem
$10^2 = 8^2 + b^2$ $a = 8$ and $c = 10$.
$100 = 64 + b^2$ Simplify.
$36 = b^2$ Subtract 64 from each side.
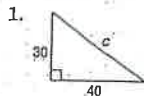$b = \pm\sqrt{36}$ Take the square root of each side.
$b = \pm 6$
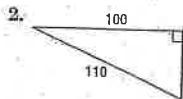
The length of the leg is 6.

Exercises

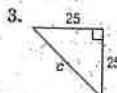Find the length of each missing side. If necessary, round to the nearest hundredth.

1. $C = 50$ (handwritten)

2. $a = 45.83$ (handwritten)

3. $C = 35.36$ (handwritten)

If c is the measure of the hypotenuse of a right triangle, find each missing measure. If necessary, round to the nearest hundredth.

4. $a = 10, b = 12, c = ?$ $C = 15.62$ (handwritten)

5. $a = 9, b = 12, c = ?$ $C = 15$ (handwritten)

6. $a = 12, b = ?, c = 16$ $b = 10.58$ (handwritten)

7. $a = ?, b = 6, c = 8$ $a = 5.29$ (handwritten)

8. $a = ?, b = \sqrt{8}, c = \sqrt{18}$ $a = 3.16$ (handwritten)

9. $a = \sqrt{5}, b = \sqrt{10}, c =$ $C = 3.87$ (handwritten)

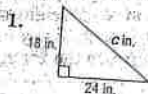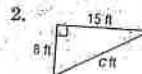## 9-4 Skills Practice
The Pythagorean Theorem

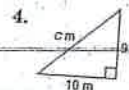Find the length of the hypotenuse in each right triangle. Round to the nearest tenth, if necessary.

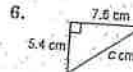1. $C = 30$ in (handwritten)

2. $C = 17$ ft (handwritten)

3. $C = 20$ yd (handwritten)

4. $C = 13.5$ ft (handwritten)

5. $C = 5.2$ m (handwritten)

6. $C = 9.3$ ft (handwritten)

If c is the measure of the hypotenuse, find each missing measure. Round to the nearest tenth, if necessary.

7. $a = ?, b = 24, c = 26$ $a = 10$ (handwritten)

8. $a = 16, b = ?, c = 34$ $b = 30$ (handwritten)

9. $a = 24, b = ?, c = 40$ $b = 32$ (handwritten)

10. $a = 5, b = ?, c = 7$ $b = 4.9$ (handwritten)

11. $a = ?, b = 32, c = 39$ $a = 22.29$ (handwritten)

12. $a = 21, b = ?, c = 48$ $b = 43.2$ (handwritten)

13. $a = 18, b = 29, c = ?$ $C = 34.13$ (handwritten)

14. $a = ?, b = 36, c = 49$ $a = 33.2$ (handwritten)

15. $a = 8, b = ?, c = 12$ $b = 8.94$ (handwritten)

16. $a = 14, b = 21, c = ?$ $C = 25.2$ (handwritten)

17. $a = ?, b = 30, c = 40$ $a = 26.46$ (handwritten)

18. $a = 4, b = ?, c = 7$ $b = 5.7$ (handwritten)

19. $a = 13, b = 18, c = ?$ $C = 22.2$ (handwritten)

20. $a = ?, b = 55, c = 75$ $a = 51.0$ (handwritten)

The lengths of three sides of a triangle are given. Determine whether each triangle is a right triangle.

21. 14 m, 5 m, 4 m   Not a rt. Δ (handwritten)

22. 3 in., 4 in., 5 in.   Yes, it is a rt. triangle (handwritten)

Close and Check

Focus Question
When you know the lengths of two sides of a right triangle, how do you find the third?

Knowing the 2 legs, use Pythagorean Theorem to find the 3rd side (hypotenuse). In a rt. triangle, the sum of the squares of the legs is equal to the square of the hypotenuse.
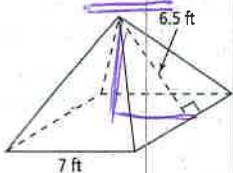
Do you know HOW?

1. Find the height of the sheet of paper. Round to the nearest whole number.

$a^2 + b^2 = c^2$
$a^2 + 11^2 = 20.2^2$

20.2 in.

11 in.
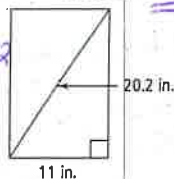
height = **17** in.

2. Find the height of the square pyramid. Round to the nearest tenth.

6.5 ft

7 ft

height = **5.5** ft

$a^2 + b^2 = c^2$
$a^2 + \left(\dfrac{7}{2}\right)^2 = 6.5^2$

Do you UNDERSTAND?

3. Reasoning A square has a diagonal of $\sqrt{50}$. How can you find the side length of the square?
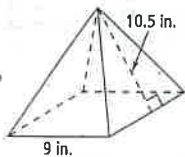
The sides are the legs, so
$a^2 + a^2 = (\sqrt{50})^2$
$2a^2 = 50$
$a^2 = 25$
$a = 5$

4. Error Analysis Your friend found the height of the square pyramid. Explain her error.

10.5 in.

$4.5^2 + 10.5^2 = c^2$
$130.5 = c^2$
$11.4 \approx c$

9 in.

Should be the hypotenuse, not a leg.

Should be...
$4.5 + b^2 = 10.5^2$

Close and Check

Focus Question
How can you determine whether a triangle is a right triangle if you do not know its angle measures and do not have measuring tools?
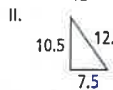
Converse of the Pythag. Theorem: If the sum of the squares of the 2 shortest side lengths of a triangle is equal to the sq. of the longest side length of the triangle, then the triangle is a rt.
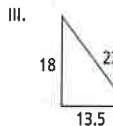
Do you know HOW?

1. Which triangles are right triangles?

I.

16   20
12

Yes

II.

10.5   12.5
7.5

No

III.

18   22.5
13.5
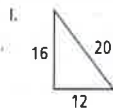
Yes

2. Write an R in the box to the right of the sides lengths that form a right triangle.

A. 30, 40, 50   **R**

B. 10, 15, 20   ☐

C. 18, 24, 30   **R**

D. 2, 2, 2.8   ☐

Do you UNDERSTAND?

3. Reasoning Your friend says that since the lengths 3, 4, and 5 form a right triangle, then the lengths 6, 8, and 10 must also form a right triangle. Explain why this works.

Yes, this works because all side lengths are twice as big.
$6^2 + 8^2 = 10^2$ ✓

$(3,4,5) \times 10$

$(3,4,5) \times 6$

4. Writing If you have two 1-inch segments, how can you choose a third segment to make a right triangle? Is there more than one choice you can make? Explain.
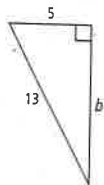
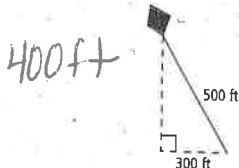$1^2 + 1^2 = c^2$ & solve for c to get $\sqrt{2}$.

WORK ATTACHED

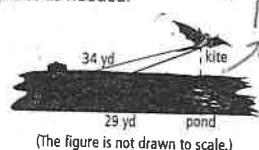**1.** Find the length of the third side of the right triangle. Simplify your answer.

b=12

2. Tyler is flying a kite on 500 feet of string. How high is it above the ground if the horizontal distance between Tyler and the kite is 300 feet? Simplify your answer.

400ft

3. You are flying your dragon kite. It's connected to 34 yd of string. The kite is directly above the edge of a pond. The edge of the pond is 29 yd from where the kite is tied to the ground. How high is the kite above the edge of the pond? Round to one decimal place as needed.

17.7yd

(The figure is not drawn to scale.)

4. Think About the Process A 10-foot ladder is leaning against a building with the base of the ladder 2 feet from the building.
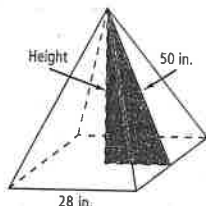
a. How can you find how high on the building the top of the ladder will reach?

A. Subtract the two given values.
B. Add the two given values.

C. Substitute the values for a and b in the equation $a^2 + b^2 = c^2$ and solve for c.

D. Substitute the values for a and c in the equation $a^2 + b^2 = c^2$ and solve for b.

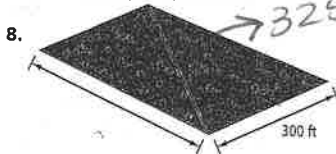b. How far up the building will the top of the ladder reach?

9.8ft

5. Find the height of the square pyramid.

Height   50 in.

48in.

28 in.

(The figure is not drawn to scale.)

6. A stainless steel patio heater is a square pyramid. The length of one side of the base is 19.8 in. The slant height of the pyramid is 92.8 in. What is the height of the pyramid? Round to one decimal place as needed.

92.3in.

7. The base of a square pyramid is 13 ft long. Its slant height is 34.2 ft. Find the height of the square pyramid. Round to one decimal place as needed.
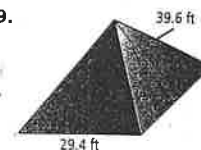
33.6ft.

8.

→325ft
125ft.

300 ft

(The figure is not drawn to scale.)

a. Writing What is the length of the rectangular plot of land shown?

b. How are the lengths of the legs of a right triangle related to the lengths of the sides of a rectangle?

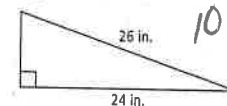The legs are the same measurements of the length & width of a square.

9.

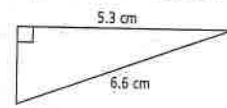39.6 ft
29.4 ft

(The figure is not drawn to scale.)

36.8ft

a. Reasoning What is the height of the square pyramid? Round to one decimal place as needed.

b. Once you know which length represents the hypotenuse, does it matter which length you substitute for a and which length you substitute for b? Explain.

No

10. Find the length of the unknown leg of the right triangle.

10in.

26 in.
24 in.

(The figure is not drawn to scale.)

11. Error Analysis A student was asked to find the length of the unknown leg of the right triangle. She incorrectly said that the length of the unknown leg of the right triangle is about 6.2 cm.

3.9cm

5.3 cm
6.6 cm

(The figure is not drawn to scale.)

a. Find the length of the unknown leg of the right triangle. Round to one decimal place as needed.

3.9cm

b. What mistake might the student have made?

She did not square the length of the given leg.

12. Camping A guy rope is attached to the top of a tent pole. The guy rope is pegged into the ground 6 feet from the tent. If the guy rope is 11 feet long, how long is the tent pole? Round to one decimal place as needed.

9.2ft.

13. The area of the base of a square pyramid is 100 cm². Its slant height is 36.8 cm. Find the height of the square pyramid. Round to one decimal place as needed.

36.5cm

14. Think About the Process A lamp base is a square pyramid. One side of the base is 13 cm long. The slant height is 35.6 cm.

a. What is the first step in finding the height of the square pyramid?

A. Substitute the given values into $a^2 + b^2 = c^2$.
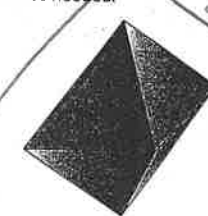
B. Divide the length of one side of the base by 2.

C. Add the two given values.
D. Subtract the two given values.

b. Find the height of the square pyramid. Round to one decimal place as needed.

35 cm

15. Challenge To get to a cabin, Diana can bike west from a parking lot along the edge of a rectangular reservoir for 0.5 miles and then south along the edge. Alternatively, she can row a boat directly from the parking lot for 1.3 miles. If Diana can ride 1.4 times as fast as she can row, which method of travel will get Diana to the cabin faster?

Biking

16. Challenge An 8-sided game piece has the shape of two identical square pyramids attached at their bases. The perimeter of the square is 80 mm and the slant height of each pyramid is 17 mm. What is the distance from the apex of one pyramid to the apex of the other? Round to one decimal place as needed.

27.4 mm

1)
$$a^2+b^2=c^2$$
$$5^2+b^2=13^2$$
$$25+b^2=169$$
$$-25 \qquad -25$$
$$b^2=144$$
$$\boxed{b=12}$$

2)
$$a^2+b^2=c^2$$
$$300^2+b^2=500^2$$
$$90000+b^2=250000$$
$$-90000 \qquad -90000$$
$$b^2=160000$$
$$\boxed{b=400\,ft}$$

3)

34, a, 29
$$a^2+29^2=34^2$$
$$a^2+841=1156$$
$$-841 \qquad -841$$
$$a^2=315$$
$$\boxed{a=17.7\,yd}$$

4)

10ft, a, 2ft
$$a^2+b^2=c^2$$
$$a^2+(2)^2=10^2$$
$$a^2+4=100$$
$$-4 \qquad -4$$
$$a^2=96$$
$$\boxed{a=9.8\,ft}$$

5)
$$a^2+b^2=c^2$$
$$\left(\frac{28}{2}\right)^2+b^2=50^2$$
$$14^2+b^2=2500$$
$$196+b^2=2500$$
$$-196 \qquad -196$$
$$b^2=2304$$
$$\boxed{b=48in}$$

6)

slant height = 92.8 in
19.8
19.8
$$a^2+b^2=c^2$$
$$\left(\frac{19.8}{2}\right)^2+b^2=92.8^2$$
$$9.9^2+b^2=8611.84$$
$$98.01+b^2=8611.84$$
$$-98.01 \qquad -98.01$$
$$b^2=8513.83$$
$$\boxed{b=92.3in}$$

8.)

A)

325 ft. (diagonal)
300 ft. (bottom)
a (side)
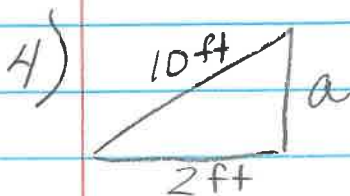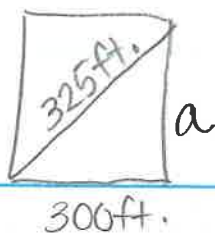
$$a^2 + b^2 = c^2$$
$$a^2 + 300^2 = 325^2$$
$$a^2 + 90{,}000 = 105{,}625$$
$$\underline{-90{,}000 \quad -90{,}000}$$
$$a^2 = 15{,}625$$
$$\boxed{a = 125\text{ ft}}$$

7.)

Slant height $= 34.2$

13
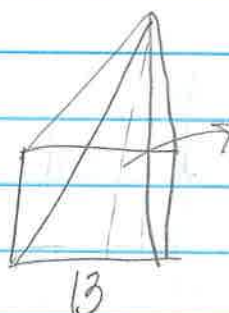
$$a^2 + b^2 = c^2$$
$$a^2 + \left(\frac{13}{2}\right)^2 = (34.2)^2$$
$$a^2 + 6.5^2 = 1{,}169.64$$
$$a^2 + 42.25 = 1{,}169.64$$
$$\underline{-42.25 \quad -42.25}$$
$$a^2 = 1{,}127.39$$
$$\boxed{a = 33.6\text{ ft}}$$

9)

$$a^2 + b^2 = c^2$$ *39.6 ft is the slant height
$$a^2 + \left(\frac{29.4}{2}\right)^2 = (39.6)^2$$
$$a^2 + 14.7^2 = 1568.16$$
$$a^2 + 216.09 = 1{,}568.16$$
$$\underline{-216.09 \quad -216.09}$$
$$a^2 = 1352.07$$
$$\boxed{a = 36.8\text{ ft}}$$

10)
$$a^2 + b^2 = c^2$$
$$a^2 + 24^2 = 26^2$$
$$a^2 + 576 = 676$$
$$ -576 \quad -576$$
$$a^2 = 100$$
$$\boxed{a = 10 \text{ in}}$$

11)
A)
$$a^2 + 5.3^2 = 6.6^2$$
$$a^2 + 28.09 = 43.56$$
$$ -28.09 \quad -28.09$$
$$a^2 = 15.47$$
$$\boxed{a = 3.9 \text{ cm}}$$

12)

11 ft Rope   Tent pole

6 ft.
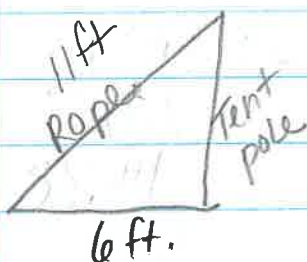
$$a^2 + b^2 = c^2$$
$$6^2 + b^2 = 11^2$$
$$36 + b^2 = 121$$
$$-36 -36$$
$$b^2 = 85$$
$$\boxed{b = 9.2 \text{ ft}}$$

13)

Slant height = 36.8 cm

$A = 100 \text{ cm}^2$

① Find the side length of the sq. base
$$A = s^2$$
$$\sqrt{100} = \sqrt{s^2}$$
$$10 = s$$

②
$$a^2 + b^2 = c^2$$
$$\left(\frac{10}{2}\right)^2 + b^2 = 36.8^2$$
$$25 + b^2 = 1,354.24$$
$$b^2 = 1,329.24$$
$$\boxed{b = 36.5 \text{ cm}}$$

14) $a^2 + b^2 = c^2$
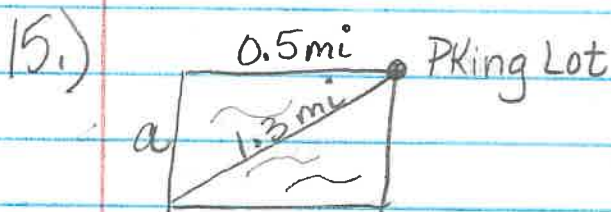
$\left(\dfrac{13}{2}\right)^2 + b^2 = 35.6^2$

$6.5^2 + b^2 = 1,267.36$

$42.25 + b^2 = 1,267.36$

$-42.25 \qquad\qquad -42.25$

$b^2 = 1,225.11$

$b = 35\,cm$

15.)

0.5mi • PKing Lot

a 1.3 mi

1) $a^2 + b^2 = c^2$

$a^2 + (0.5)^2 = (1.3)^2$

$a^2 + 0.25 = 1.69$

$\qquad -0.25 \quad -0.25$

$a^2 = 1.44$

$a = 1.2\,mi$

← WEST

2) $D = R \cdot T$

Bike Distance = $1.4x$

$(0.5 + 1.2) = 1.4x$

$\dfrac{1.7}{1.4} = \dfrac{1.4x}{1.4}$

$x = 1.214$ time $\Longrightarrow$ SHORTER amt. of time (exact unit of time not provided in problem)

3) $D = R \cdot T$

Boat Distance = $1x$

$1.3 = 1x$

$1.3 = x$